

5.4 Integrals and the Net Change Theorem

- $\int f(x) dx$ is called an indefinite integral that is equivalent to finding an antiderivative. Thus, we can see the following;

$$\int f(x) dx = F(x) \Rightarrow F'(x) = f(x).$$

- Have a look at the formula for indefinite integrals.

Example1

Find the indefinite integral.

$$\int (x^3 - \sec^2 x) dx.$$

Example2

Verify by differentiation that the formula is correct.

$$\int \frac{x}{\sqrt{x^2 + 1}} dx = \sqrt{x^2 + 1} + C.$$

The Net Change Theorem

Example 3

Evaluate the following definite integral.

$$\int_{-1}^2 2|x| dx.$$

- The **Net Change Theorem** (reformulation of FTC2)

$$\int_a^b F'(x) dx = \int_a^b \frac{dF(x)}{dx} dx = F(b) - F(a).$$

- For the position function $s(t)$ of the particle, its velocity is $s'(t) = v(t)$. So by the **NCT** its displacement from t_1 to t_2 will be

$$s(t_2) - s(t_1) = \int_{t_1}^{t_2} v(t) dt.$$

The total distance traveled will be

$$\int_{t_1}^{t_2} |v(t)| dt.$$

Example4

A particle moves along a line so that its velocity at time t is $v(t) = 2t - 4$ (measured in meters per second).

- 1 Find the displacement of the particle during the time period $0 \leq t \leq 3$
- 2 Find the distance traveled during this time period.