7.1 Integration by parts

- **The Chain Rule** for differentiation $\Rightarrow$ **the Substitution Rule** for integration.
- **The Product Rule** for differentiation $\Rightarrow$ **Integration by parts** for integration.
- **Indefinite Integrals**
  The formula for integration by parts is derived from the Product Rule

$$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx.$$

  Alternatively we may use the following formula by using the substitution ($u = f(x)$ and $v = g(x)$).

$$\int u\,dv = uv - \int v\,du.$$

Integration by parts for definite integrals

- **For definite integrals**

$$\int_a^b f(x)g'(x)dx = f(x)g(x)|_a^b - \int_a^b f'(x)g(x)dx.$$

Example1

1 Evaluate

$$\int \ln x \, dx.$$

2. Evaluate

$$\int_1^e \frac{\ln x}{x^2} dx.$$

Tabular Integration

- The tabular integration is easier than integration by parts. Consider the following integral

$$\int f(x)\,g(x)\,dx.$$

If f can be differentiated repeatedly to become zero (usually f can be a polynomial) and g can be integrated repeatedly without difficulty(usually g can be a sine or cosine or exponential function), we can use the tabular integration.

Example2

Use the tabular integration to evaluate the following integral

$$\int x^2 e^x\,dx.$$

Tabular Integration Cont'd

Example3

1. Use the tabular integration to evaluate the following integral

$$\int_0^{\pi/2} t^3 \sin 2t \, dt.$$

2. First make a substitution and then use integration by parts to evaluate the following integral

$$\int_0^{\sqrt{\pi}} \theta^3 \cos\left(\theta^2\right) d\theta.$$