7.2 Trigonometric Integrals

- In this section, our goal is to use several trigonometric identities to learn how to integrate certain combinations of only "sine" and "cosine".

Example1

Using the Pythagorean identity $\sin^2 x + \cos^2 x = 1$, evaluate the following integrals:

1. $$\int \sin^3 x \, dx.$$

2. $$\int_0^{\pi/4} \sin^3 x \cos x dx.$$

Trigonometric Integrals Cont'd

Example2

1. Using the double(half) angle identity, evaluate

$$\int_0^{\pi/2} \sin^2 x \, dx.$$

2. Evaluate

$$\int_0^{\pi/4} \cos 5x \cos 2x dx.$$

- Note that

$$\int f(x)g(x)dx \neq \int f(x)dx \int g(x)dx.$$

When we have the product of sin or cosine, use the following identities:

1. $\sin A \cos B = \frac{1}{2}(\sin(A-B)+\sin(A+B))$
2. $\sin A \sin B = \frac{1}{2}(\cos(A-B)-\cos(A+B))$
3. $\cos A \cos B = \frac{1}{2}(\cos(A-B)+\cos(A+B))$

The average value of functions

- The average value of the function $f(x)$ on the interval $[a, b]$ is

$$\frac{1}{b-a}\int_a^b f(x)dx.$$

Example3

Find the average of the function $f(x) = \cos^2 x \tan^3 x$ on the interval $[0, \pi/4]$.
(Hint: Change $\tan x$ into $\sin x/\cos x$)