

## 7.3 Trigonometric Substitution

- Three basic substitutions (for  $a > 0$ )
- ① If an integrand contains the type of  $a^2 - x^2$ , substitute  $x = a \sin \theta$  with  $-\pi/2 \leq \theta \leq \pi/2$ .
- ② If an integrand contains the type of  $a^2 + x^2$ , substitute  $x = a \tan \theta$  with  $-\pi/2 < \theta < \pi/2$ .
- ③ If an integrand contains the type of  $x^2 - a^2$ , substitute  $x = a \sec \theta$  with

$$\begin{cases} 0 \leq \theta < \pi/2 & \text{if } x/a \geq 1 \\ \pi \leq \theta < 3\pi/2 & \text{if } x/a \leq -1 \end{cases}$$

- Note that we will not do the third type in our class, because the idea is pretty much same as the first and second type but is more complicated.

## Examples

1. Show that the area of a circle is  $\pi r^2$ , where its radius is  $r$ .
2. Evaluate

$$\int_{-1}^1 \sqrt{1-x^2} dx.$$

Note that there is a simple way to evaluate the definite integral.

3. Evaluate

$$\int \frac{dx}{\sqrt{1+x^2}}.$$

4. Evaluate

$$\int \sqrt{2x-x^2} dx.$$

(Hint: you need to get a perfect square inside of the radical.)