- Areas between curves

1. We consider the region S that lies between two curves $y = f(x)$ and $y = g(x)$ and two vertical lines $x = a$, $x = b$, where $f(x) \geq g(x)$ for all $x \in [a, b]$.
2. By using the **Riemann sums,** we can define the area A of the region S:

$$A = \lim_{n \to \infty} \sum_{i=1}^{n} [f(x_i^*) - g(x_i^*)] \Delta x.$$

3. We draw a picture to understand to the previous formula.
4. The area A of the region bounded by the curves $y = f(x)$, $y = g(x)$, and the vertical lines $x = a$, $x = b$ is

$$A = \int_a^b (f(x) - g(x))\, dx,$$

where f and g are continuous and $f(x) \geq g(x)$ for all $x \in [a, b]$.

- How do we find the area between the curves $y = f(x)$ and $y = g(x)$, where $f(x) \geq g(x)$ for some values of x but $g(x) \geq f(x)$ for other values of x?
- We will use the following formula: The area A between the curves $y = f(x)$ and $y = g(x)$ and between $x = a$ and $x = b$ is

$$A = \int_a^b |f(x) - g(x)| \, dx.$$

If a region is bounded by the curves $x = f(y)$, $x = g(y)$, and the horizontal lines $y = c$, $y = d$, then its area A is

$$A = \int_c^d |f(y) - g(y)| \, dy.$$