6 Applications of Integration

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Outline of Chapter 6

- Areas between curves
- Volumes
- Volumes by Cylindrical Shells

6.1 Areas between curves

- So far we have studied several techniques to find out antiderivatives:

1. Substitution (Trigonometric Substitution)
2. Integration parts (The Tabular Integration)
3. Partial Fraction Decomposition

- In this section, our goal is to set up integrals to find **areas of regions** that lie between the graphs of two continuous functions

- We consider the region S that lies between two curves $y = f(x)$ and $y = g(x)$ and two vertical lines $x = a$, $x = b$, where $f(x) \geq g(x)$ for all $x \in [a, b]$.
- By the **Riemann sum**(p.366) we can define the area A of the region S:

$$A = \lim_{n \to \infty} \sum_{i=1}^{n} \left[f\left(x_i^*\right) - g\left(x_i^*\right) \right] \Delta x.$$

- We draw a picture to understand to the previous formula.
- The area A of the region bounded by the curves $y = f(x)$, $y = g(x)$, and the vertical lines $x = a$, $x = b$ is

$$A = \int_a^b \left(f(x) - g(x) \right) dx,$$

where f and g are continuous and $f(x) \geq g(x)$ for all $x \in [a, b]$.

Example1

1. Find the area of the region bounded above by $y = e^{2x}$, bounded below by $x-$axis, and bounded on the sides $x = 0$ and $x = 2$.
2. Find the area of the region bounded above by $y = e^{x}$, bounded below by $y = x/2$, and bounded on the sides $x = 0$ and $x = 1$.
3 Find the area of the region enclosed by the parabolas $y = x^2$ and $y = -x^2 + 4x$.
4 Find the area of the region bounded by $y = x/\sqrt{1-x^2}$and $x-$axis, and bounded on the sides $x = 0$ and $x = 1/2$.
5. Find the area of the region bounded by curves $y = 1/\left(x^2+1\right)$ and $x-$axis, and bounded on the sides $x = 0$ and $x = 1$.

- How do we find the area between the curves $y = f(x)$ and $y = g(x)$, where $f(x) \geq g(x)$ for some values of x but $g(x) \geq f(x)$ for other values of x?
- We will use the following formula: The area A between the curves $y = f(x)$ and $y = g(x)$ and between $x = a$ and $x = b$ is

$$A = \int_a^b |f(x) - g(x)|\, dx.$$

Example2

Find the area of the region bounded by curves $y = \sin x$ and $y = \cos x$, and $x = 0$ and $x = \pi/2$.

- If a region is bounded by the curves $x = f(y)$, $x = g(y)$, and the horizontal lines $y = c$, $y = d$, then its area A is

$$A = \int_c^d |f(y) - g(y)|\, dy.$$

Example3

Find the area enclosed by the line $y = x - 1$ and the parabola $y^2 = x + 1$.