- Consider the following two examples:

1. Can we find the volume of the solid obtained by rotating about the y-axis the region bounded by $y = -x^3 + x$ and x-axis? Yes, we can. However it is not easy, because we need to solve the equation $y = x^3 + x$ for x in terms of y.
2. Can we find the volume of the solid obtained by rotating about the y-axis the region bounded by $y = \frac{\sin x}{x}$ and x-axis, and two vertical lines $x = \pi/6$, $x = \pi/3$? No, we cannot, because it is impossible to solve the equation $y = \frac{\sin x}{x}$ for x in terms of y.

- Draw the pictures for the two examples!
- You will see that the it is extremely hard to use disk(washer) method. The shell method is easier to use in those cases.
- The remarkable difference from the disk(washer method) is to consider the volume of **cylindrical shells**.

- A good approximation to the volume V of the solid S is given by the sum of the volumes of the shells:

$$V \simeq \sum_{i=1}^{n} 2\pi \bar{x}_i f(\bar{x}_i)\, \Delta x.$$

- The volume V can be estimated through a limiting process.

1. The volume V of the solid obtained by rotating about the y-axis the region under the curve $y = f(x)$ from $x = a$ to $x = b$ is

$$V = \int_a^b 2\pi x\, f(x) dx, \text{ where } 0 \leq a < b.$$

2. The volume V of the solid obtained by rotating about the x-axis the region under the curve $x = g(y)$ from $y = c$ to $y = d$ is

$$V = \int_c^d 2\pi y\, g(y) dy, \text{ where } 0 \leq c < d.$$

Examples

1. Find the volume of the solid obtained by rotating about y-axis the region bounded by $f(x) = -x^3 + x$ for $x \geq 0$ and $y = 0$.
2. Consider the following function

$$f(x) = \frac{\sin x}{x}, \quad \frac{\pi}{6} \leq x \leq \pi.$$

Find the volume of the solid generated by revolving about the $y-$axis the region between $y = f(x)$ and x-axis.
3. Find the volume of the solid generated by revolving about the y-axis the region bounded by $y = x^2 e^{2x}$ and $y = 0$ and $x = 1$.
4. Find the volume of the solid obtained by rotating about the y-axis the region between $y = x$ and $y = x^2$. (Use both the washer method and shell method)
5.Find the volume of the solid obtained by rotating about the x-axis the region under the curve $y = 2\sqrt{x}$ from 0 to 1.
6. Find the volume of the solid obtained by rotating the region bounded by $y = -x^2 + x$ and $y = 0$ about the line $x = 3$.