8.1 Arc Length

- In order to find the length of a curve C defined by the equation $y = f(x)$, we consider a polygonal approximation to the curve by partitioning the interval $[a, b]$, where f is continuous on $[a, b]$.
- We note that the procedure for defining arc length is very similar to the procedure we used for defining area and volume.
- By applying the MVT to f on the subinterval $[x_{i-1}, x_i]$, we see that there is number $x_i^* \in [x_{i-1}, x_i]$ such that

$$f(x_i) - f(x_{i-1}) = f(x_i^*)(x_i - x_{i-1}).$$

- Through a limiting process, we obtain the following arc length formula: if $f'$is continuous on $[a, b]$, then the length of the curve $y = f(x)$ on $[a, b]$ is

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx.$$

Example1

Find the length of the arc length of the semicubical parabola $y^2 = x^3$ between the points $(1, 1)$ and $(9, 27)$.

- For the equation $x = g(y)$, with $c \leq y \leq d$,

$$L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy.$$

Example2

Find the length of the arc curve of the parabola $y^2/2 = x$ from $(0, 0)$ to $(\sqrt{2}, 1)$.

- The surface area of the surface obtained by rotating the curve $y = f(x)$, $a \leq x \leq b$, about x-axis is

$$S = \int_a^b 2\pi f(x) \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx.$$

Example1

Consider The curve $y = \sqrt{9 - x^2}$ over $-1 \leq x \leq 1$. Find the area of the surface obtained by rotating this arc about the x-axis.

- The surface area of the surface obtained by rotating the curve $x = g(y)$, $c \leq y \leq d$, about x-axis is

$$S = \int_c^d 2\pi y \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

Example2

The arc of the parabola $y = x^2$ from $(0, 0)$ to $(1, 1)$ is rotated about the y-axis. Find the area of the resulting surface.