10 Parametric Equations and Polar Coordinates

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Outline of Chapter 10

- Curves Defined by Parametric Equations
- Calculus with Parametric Curves
- Polar Coordinates
- Areas and Length in Polar Coordinates

- When you consider a particle which moves along a curve, considering parametric equations is a better way to decribe the curve.
- We suppose that x and y are both given as functions of a third variable t (parameter) by the Parametric Equations

$$x = f(t), \quad y = g(t)$$

Each paramter t determines a point $(x, y) = (f(t), g(t))$ which traces out a parameter curve C as t varies.

Example1

The following parametric equations

$$x(t) = t^2 - 3t \quad y(t) = t - 1 \quad \text{for all real numbers } t$$

will be the parabola(Cartesian equation) $x = y^2 - y - 2$.

- We can restrict t $(a \leq t \leq b)$ to consider parmetric equations

$$x = f(t) \quad y = g(t) \quad a \leq t \leq b,$$

where $(f(a), g(a))$ is an initial point and $(f(b), g(b))$ is a terminal point.

Example2

1. Find the curve represented by the following parametric equations

$$x = \cos t \quad y = \sin t \quad 0 \leq t \leq 2\pi.$$

2. Find the curve represented by the following parametric equations

$$x = 3\cos 4t \quad y = 3\sin 4t \quad 0 \leq t \leq 2\pi.$$

3. We can see that $(x-h)^2 + (y-k)^2 = r^2$ (the equation of circles) is equivalent to the following parametric equations

$$x = h + r\cos t \quad y = k + r\sin t \quad 0 \leq t \leq 2\pi.$$

Example3

Sketch the curve with the parametric equations

$$x = \cos t \quad y = \cos^2 t$$

Example4

Eliminate the parameter to find a Cartesian equation of the curve.

1. $x = 5t - 3 \quad y = 2t - 1$
2. $x = \sqrt{t} \quad y = t - 1$
3. $x = e^{3t} \quad y = t + 2 \quad t \geq -2$

Example5

Describe the motion of a particle with position (x, y) as t varies in the given interval

$$x = 1 + 2\cos t \quad y = -2 + 2\sin t, \quad \pi/2 \leq t \leq 3\pi/2$$