- Tangents

  Consider the curve $y = F(x)$ with the parametric equations $x = f(t)$ and $y = g(t)$. By substitution, we can obtain

  $$g(t) = F(f(t)).$$

  Then the Chain Rule gives

  $$g'(t) = F'(f(t))f'(t) = F'(x)f'(t).$$

  Thus the slope of the tangent to the curve $y = F(x)$ at $(x, F(x))$ is

  $$F'(x) = \frac{g'(t)}{f'(t)} \text{ or } \frac{dy}{dx} = \frac{dy/dt}{dx/dt} \text{ if } \frac{dx}{dt} \neq 0$$

- Note that the second derivative d^2y/dx^2 is

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

Example1

A curve C is defined by the parametric equations $x = t^2$, $y = t^3 - t$.
1. Show that C has two tangents at the point $(1, 0)$ and find their equations
2. Find the points on C where the tangent is horizontal or vertical
3. Determine where the curve is concave upward or downward

Example2

Find an equation of the tangent to the curve at the point corresponding to the given value of the parameter.

$$x = e^{\sqrt{t}}, \quad y = t - \ln t^2; \quad t = 1$$

Example3

Find an equation of the tangent to the curve at the given point by two methods:

1. without eliminating the parameter
2. by the first eliminating the parameter

$$x = 1 + \ln t, \quad y = t^2 + 1; \quad (1,2)$$

Example4

Find dy/dx and d^2y/dx^2. For which values of t is the curve concave upward?

$$x = 3\sin t, \quad y = 2\cos t, \quad 0 < t < 2\pi$$

Example5

Find the points on the curve where the tangent line is horizontal or vertical.

$$x = 2\cos\theta, \quad y = \sin 2\theta$$