

10.5 The Ratio and Root Tests

• Ratio Test

1. If

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = L < 1,$$

then $\sum_{n=1}^{\infty} a_n$ is (absolutely) convergent.

2. If

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = L > 1 \text{ or } \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \infty,$$

then $\sum_{n=1}^{\infty} a_n$ is divergent.

3. If

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = 1,$$

the Ratio Test is inconclusive.

• Root Test

1. If

$$\lim_{n \rightarrow \infty} |a_n|^{1/n} = L < 1,$$

then $\sum_{n=1}^{\infty} a_n$ is (absolutely) convergent.

2. If

$$\lim_{n \rightarrow \infty} |a_n|^{1/n} = L > 1 \text{ or } \lim_{n \rightarrow \infty} |a_n|^{1/n} = \infty,$$

then $\sum_{n=1}^{\infty} a_n$ is divergent.

3. If

$$\lim_{n \rightarrow \infty} |a_n|^{1/n} = 1,$$

the Root Test is inconclusive.