- The Convergence Tests(Integral test and two comparison tests) that we have considered so far apply only to series

$$\sum_{n=1}^{\infty} a_n, \quad \text{with } a_n \geq 0.$$

- In this section, we will deal with series whose terms alternate in sign.
- nth term of an alternating series $\sum a_n$ is of the form

$$a_n = (-1)^{n-1} b_n, \quad a_n = (-1)^n b_n,$$

where $b_n \geq 0$ for $n \geq 1$.
- Examples of alternating series

$$-1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} + \cdots = \sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$$

$$\frac{1}{2} - \frac{2}{3} + \frac{3}{4} - \frac{4}{5} \cdots = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n+1}$$

The Alternating Series Test

If the alternating series

$$\sum_{n=1}^{\infty}(-1)^{n+1}b_n = b_1 - b_2 + b_3 - b_4 + b_5 - b_6 + \cdots, \quad b_n > 0$$

satisfies

$$(1)\, b_{n+1} \le b_n \quad \text{for all } n \ge 1$$
$$(2)\, \lim_{n\to\infty} b_n = 0,$$

then the series converges.

Example1

The following alternating $p-$series is convergent by the alternating series test:

$$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n^p} \quad \text{for } p > 0.$$

Example 2

Test the series for convergence or divergence.

1.
$$\frac{4}{7} - \frac{4}{8} + \frac{4}{9} - \frac{4}{10} + \cdots$$

2.
$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{2n^2+3}$$

3.
$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{7^n}$$

4.
$$\sum_{n=1}^{\infty} \frac{\sin(n\pi - \pi/2)}{n^{2/3}}$$

5.
$$\sum_{n=1}^{\infty} (-1)^{n} \frac{n^n}{n!}$$