- Our goal is to learn how to express a power series representation for certain types of functions by manupulating **geometric series** or by differentiating or integrating such a series.
- We recall the geometric series

$$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + \cdots x^n + \cdots = \frac{1}{1-x}, \quad \text{for } |x| < 1.$$

Example1

1. Express $1/\left(1-x^2\right)$ as the sum of a power series and find the interval of convergence.
2. Find a power series representation for $2/(x+3)$ and find the interval of convergence.
3. Find a power series representation for $x/\left(4+x^2\right)$ and find the interval of convergence.

Theorem

If $\sum_{n=0}^{\infty} c_n(x-a)^n$ *has the radius of convergence* $R > 0$*, then*
1. the function f *is differentiable on the interval* $(a-R, a+R)$*:*

$$f(x) := \sum_{n=0}^{\infty} c_n(x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + \cdots$$

2.

$$\frac{d}{dx}\left[\sum_{n=0}^{\infty} c_n(x-a)^n\right] = \sum_{n=0}^{\infty}\left[\frac{d}{dx} c_n(x-a)^n\right]$$

3.

$$\int\left[\sum_{n=0}^{\infty} c_n(x-a)^n\right]dx = \sum_{n=0}^{\infty}\int c_n(x-a)^n dx$$

The radii of conv. of the power series in eqs. 2 and 3 are both R*.*

- Note that both sides in eqs. of 2 and 3 may not be equal for other types of series and that the interval of convergence for 2 and 3 may be different from 1.

Example2

1. Consider the geometric series $\sum_{n=0}^{\infty} x^n = 1/(1-x) \quad \text{for } |x| < 1.$ Express $1/(1-x)^2$ as a power series. Find R.

2. Find a power series represenation for $\ln(1-x)$ and its radius of convergence. Express $\ln(\frac{1}{2})$ as an infinite series.

3. Find a power series representation for $f(x) = \tan^{-1} x$. Find R.

4. Evaluate the indefinite integral as a power series. What is the radius of convergence?

$$(1) \quad \int \frac{1}{1-x^{10}}\, dx$$

$$(2) \quad \int \frac{x - \tan^{-1} x}{x^2}\, dx$$