11.5 Describing Lines and Planes

- **Vector Equation for a line**

  Vector Equation of a line L through $P_0(x_0, y_0, z_0)$ parallel to v:

  $$\mathrm{r}(t) = \mathrm{r}_0 + t\mathrm{v} \text{ or } \mathrm{r}_0 + \frac{t}{|\mathrm{v}|}\mathrm{v}, \quad -\infty < t < \infty,$$

  where r is the position vector of a point $P(x, y, z)$ on L and r_0 is the position vector of $P_0(x_0, y_0, z_0)$.

- **Parametric Equation for a Line**

  The standard parametrization of the line through $P_0(x_0, y_0, z_0)$ parallel to $\mathrm{v} = \langle v_1, v_2, v_3 \rangle$ is

  $$x = x_0 + tv_1, \quad y = y_0 + tv_2, \quad z = z_0 + tv_3, \quad -\infty < t < \infty.$$

- **Symmetric Equations** for a Line

  $$\frac{x - x_0}{v_1} = \frac{y - y_0}{v_2} = \frac{z - z_0}{v_3},$$

  where $v_1 \neq 0$, $v_2 \neq 0$ $v_3 \neq 0$.

- **Line segment** from r_0 to r_1 is given by the vector equation

  $$\mathrm{r}(t) = \mathrm{r}_0 + t(\mathrm{r}_1 - \mathrm{r}_0) = (1 - t)\mathrm{r}_0 + t\mathrm{r}_1, \quad 0 \leq t \leq 1.$$

Example1

1. Find a vector equation and parametric equations for the line that passes through the point $(-1,0,3)$ and is parallel to the vector $\mathrm{v} = 2\mathrm{i} - 3\mathrm{j} + \mathrm{k}$. Find two other points on the line.
2. Find parametric equations and symmetric equations for the line through $P(-3,2,-1)$ and $Q(-1,1,4)$.
3. Find a vector equation and parametric equations for line segment from $(2,-1,3)$ to $(3,5,1)$.
4. Show that the lines L_1 and L_2 with parametric equations are parallel.
5. Show that the lines L_2 and L_3 with parametric equations are skew.

$$L_1 : x = -6t, \quad y = 1 + 9t, \quad z = -3t$$
$$L_2 : x = 1 + 2s, \quad y = 4 - 3s, \quad z = s$$
$$L_3 : x = -2t, \quad y = 1 + 2t, \quad z = -3t$$

- **Equations for a Plane**
  For arbitrary points $P(x,y,z)$ the plane through $P_0(x_0,y_0,z_0)$ normal to $\mathrm{n} = A\mathrm{i} + B\mathrm{j} + C\mathrm{k}$ has

1. **Vector equation**: $\mathrm{n} \cdot \overrightarrow{P_0P} = 0$
2. **Component equation**:

$$A(x-x_0) + B(y-y_0) + C(z-z_0) = 0$$

- Component equation simplified: $Ax + By + Cz = D$ with $D = A_0 + B_0 + C_0$.

Example2

1. Find an equation for the plane through $P_0(-2,1,5)$ perpendicular to $\mathrm{n} = 4\mathrm{i} + \mathrm{j} - 3\mathrm{k}$.
2. Find an equation for the plane through $A(0,0,1)$, $B(3,0,0)$, and $C(0,2,0)$.