12 Vectors and the Geometric of Space

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Outline of Chapter 12

1. Three-Dimensional Coordinates Systems
2. Vectors
3. The Dot Product
4. The Cross Product
5. Equations of Lines and Planes

12.1 Three-Dimensional Coordinates Systems

- The coordinates in a three dimensional system are of the form (x, y, z), called the ordered triple.
- A point P is located at $(x, y, z) \Rightarrow P(x, y, z)$.
- The Cartesian product $\mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R} = \{(x, y, z) | x, y, z \in \mathbb{R}\}$

Example1

Describe the region of $\mathbb{R}^3$ represented by the equation or inequality.

1. $z = 1$
2. $x^2 + y^2 + z^2 \leq 4$

- The Distance between $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is

$$\left|\overline{P_1 P_2}\right| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}.$$

Example2

Find the distance between $P_1(1, 2, 3)$ and $P_2(-3, -1, 0)$.

- The Standard Equation for a Sphere with Radius r and Center (x_0, y_0, z_0) is

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$$

Example3

Find the center and radius of the sphere.

1 $x^2 + y^2 + z^2 + 4x - 6z + 3 = 0$

2. $2x^2 + 2y^2 + 2z^2 = -8x + 24y + 2$