- **Vector Equation for a line**
  Vector Equation of a line L through $P_0(x_0, y_0, z_0)$ parallel to $\mathbf{v}$:
  $$\mathbf{r}(t) = \mathbf{r}_0 + t\mathbf{v} \text{ or } \mathbf{r}_0 + \frac{t}{|\mathbf{v}|}\mathbf{v}, \quad -\infty < t < \infty,$$
  where $\mathbf{r}$ is the position vector of a point $P(x, y, z)$ on L and $\mathbf{r}_0$ is the position vector of $P_0(x_0, y_0, z_0)$.
- **Parametric Equation for a Line**
  The standard parametrization of the line through $P_0(x_0, y_0, z_0)$ parallel to $\mathbf{v} = \langle v_1, v_2, v_3\rangle$ is
  $x = x_0 + tv_1, \quad y = y_0 + tv_2, \quad z = z_0 + tv_3, \quad -\infty < t < \infty.$
- **Symmetric Equations** for a Line
  $$\frac{x - x_0}{v_1} = \frac{y - y_0}{v_2} = \frac{z - z_0}{v_3},$$
  where $v_1 \neq 0$, $v_2 \neq 0$ $v_3 \neq 0$.
- **Line segment** from $\mathbf{r}_0$ to $\mathbf{r}_1$ is given by the vector equation
  $$\mathbf{r}(t) = \mathbf{r}_0 + t(\mathbf{r}_1 - \mathbf{r}_0) = (1 - t)\mathbf{r}_0 + t\mathbf{r}_1, \quad 0 \leq t \leq 1.$$

Example1

1. Find a vector equation and parametric equations for the line that passes through the point $(-1, 0, 3)$ and is parallel to the vector $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$. Find two other points on the line.
2. Find parametric equations and symmetric equations for the line through $P(-3, 2, -1)$ and $Q(-1, 1, 4)$.
3. Find a vector equation and parametric equations for line segment from $(2, -1, 3)$ to $(3, 5, 1)$.
4. Show that the lines L_1 and L_2 with parametric equations are parallel.
5. Show that the lines L_2 and L_3 with parametric equations are skew.

$$
\begin{aligned}
L_1 &: x = -6t, \quad y = 1 + 9t, \quad z = -3t \\
L_2 &: x = 1 + 2s, \quad y = 4 - 3s, \quad z = s \\
L_3 &: x = -2t, \quad y = 1 + 2t, \quad z = -3t
\end{aligned}
$$

- **Equations for a Plane**
  For arbitray points $P(x,y,z)$ the plane through $P_0\,(x_0,y_0,z_0)$ normal to $\mathbf{n}=A\mathbf{i}+B\mathbf{j}+C\mathbf{k}$ has

1. **Vector equation**: $\mathbf{n}\cdot\overrightarrow{P_0P}=0$
2. **Component equation**:

$$A\,(x-x_0)+B\,(y-y_0)+C\,(z-z_0)=0$$

- Component equation simplified: $Ax+By+Cz=D$ with $D=A\,x_0+B\,y_0+C\,z_0$.

Example2

1. Find an equation for the plane through $P_0(-2,1,5)$ perpendicular to $\mathbf{n}=4\mathbf{i}+\mathbf{j}-3\mathbf{k}$.
2. Find an equation for the plane through $A(0,0,1)$, $B(3,0,0)$, and $C(0,2,0)$.

Example3

1. Find parametric equations for the line in which the planes $3x-6y-2z=15$ and $2x+y-2z=5$ intersect. Find the angle between those planes.

2. Show that a formula for the distance D from a point $P(x_1, y_1, z_1)$ to the plane $ax+by+cz+d=0$ is

$$D=\frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}.$$