13 Vector Functions

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Outline of Chapter 13

- Consider a particle (body) moving through three dimensional space during a **time** interval I. Then how do we describe the particle(body)'s motion mathematically?
- For a time interval I, a particle's coordinates are considered as functions:

  $$x = f(t), \quad y = g(t), \quad z = h(t) \quad \text{for } t \in I.$$

  So the points $P(x, y, z) = (f(t), g(t), h(t))$ can be represented in vector form:

  $$\mathbf{u}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k} = \langle f(t), g(t), h(t) \rangle,$$

  where f, g, h are real functions called the **component functions**.
- Vector functions cause vector fields which are important to the study of the flow of fluid, gravitational fields, and electromagnetic phenomena.

Example1

If the vector function $\mathbf{r}(t)$ is defined as

$$\mathbf{r}(t) = \left\langle \frac{t}{t^2+1}, \sqrt{2t-3}, \ln(2-t) \right\rangle,$$

then find the component functions and the domain of $\mathbf{r}(t)$.

Theorem

Limits of Vector functions:

If $\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k} = \langle f(t), g(t), h(t)\rangle$, *then*

$$\begin{aligned} \lim_{t\to a} \mathbf{r}(t) &= \left(\lim_{t\to a} f(t)\right)\mathbf{i} + \left(\lim_{t\to a} g(t)\right)\mathbf{j} + \left(\lim_{t\to a} h(t)\right)\mathbf{k} \\ &= \left\langle \lim_{t\to a} f(t), \lim_{t\to a} g(t), \lim_{t\to a} h(t) \right\rangle, \end{aligned}$$

provided the limits of the component functions exist.

Example2:

1. If $\mathbf{r}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j} + t\mathbf{k}$, find

$$\lim_{t \to \pi/6} \mathbf{r}(t).$$

2. If $\mathbf{r}(t) = \mathbf{i} + e^{-t}\mathbf{j} + (\sin t/t)\mathbf{k}$, find

$$\lim_{t \to 0} \mathbf{r}(t).$$

Definition

Continuous at a Point: A vector function $\mathbf{u}(t)$ is continuous at a point $t = a$ in an interval I if $\lim_{t \to a} \mathbf{u}(t) = \mathbf{u}(a)$. The vector function is continuous if it is continuous at all $t \in I$.

- Note that $\mathbf{u}(t)$ is continuous at $t = a \Leftrightarrow$ each component function is continuous at $t = a$.

- $C = \{(x, y, z) | x = f(t), y = g(t), z = h(t)$ for $t \in [a, b]\}$ is called a **space curve**. So C is traced out by a moving particle whose position is $(f(t), g(t), h(t))$ at all $t \in [a, b]$.
- It is not easy to sketch curves for the given vector equations in $3-$dimensional coordinate systems. So H.W #7,9 are optional!

Example3:

1. Find a vector equation and parametric equations for the line segment that joins P to Q: $P(1, -1, 2), Q(6, -2, 1)$.
2. Find a vector function that represents the curve of the intersection of the cylinder $x^2 + y^2 = 4$ and the plane $y + z = 2$.