

13.4 Motion in Space; Velocity and Acceleration

Definition

Velocity, Speed, Acceleration

If \mathbf{u} is the standard position vector of a particle moving along a smooth curve in space

1. The particle's velocity is $\mathbf{v}(t) = \frac{d\mathbf{u}}{dt}$ which is tangent to the curve.
2. Speed of a particle is $|\mathbf{v}(t)| = |\mathbf{u}'(t)|$.
3. The unit vector $\mathbf{v}/|\mathbf{v}|$ is the direction of motion at time t .
4. Acceleration is $\mathbf{a}(t) = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{u}}{dt^2}$.

Example

1. Find the velocity, speed, and acceleration of a particle with position vector $\mathbf{u}(t) = \sin t \mathbf{i} + 2 \cos t \mathbf{j} + t^2 \mathbf{k}$.
2. A moving particle starts at an initial position $\mathbf{u}(0) = \langle 0, 1, 0 \rangle$ with the initial velocity $\mathbf{v}(0) = \langle -1, 2, -3 \rangle$. Its acceleration is $\mathbf{a}(t) = \langle 2t, 3t^2, 1 \rangle$. Find its velocity and position at time t .