13 Partial Derivatives

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Outline of Chapter 13

1. Functions of Several Variables
2. Limits and Continuity of Multivariable functions
3. Partial Derivatives
4. The Chain Rule for Multivariable functions
5. Directional Derivatives and the Gradient Vector
6. Tangent Planes and Differentials
7. Extreme Values of Functions of Two Variables
8. Lagrange Mutipliers

Definition

A function with more than two variables is a rule that assigns to each ordered pair $(x_1, x_2, \cdots, x_n)$ in a set D a unique real number denoted by $f(x_1, x_2, \cdots, x_n)$. D is called the domain of f and its range is $\{f(x_1, x_2, \cdots, x_n) \mid (x_1, x_2, \cdots, x_n) \in D\}$.

Example1

For each of the following functions with two variables, evaluate $f(2,1)$ and find the domain.

(1)

$$f(x,y) = \frac{\sqrt{x+y-1}}{x^2+y^2+1}$$

(2)

$$f(x,y) = y \ln(e^x + y^2)$$

Definition

If f is a function with more than two variables with domain D, then the graph of f is the set of all points $(x_1, x_2, \cdots, x_n) \in \mathbb{R}^n$ such that $x_n = f(x_1, x_2, \cdots, x_{n-1})$ and $(x_1, x_2, \cdots, x_{n-1}) \in D$.

Example2

The surface of the half sphere $g(x, y) = \sqrt{1 - x^2 - y^2}$ is a good example of graph.

Definition

The level curves of a function of more than two variables are the curves with equations $f(x_1, x_2, \cdots, x_n) = k$, where k is a constant.

Example3

Sketch the contour map of the function $g(x, y) = \sqrt{1 - x^2 - y^2}$ for $k = 0, 3/4, 1$.