13.6 Tangent Planes and Differentials

- Suppose that f is a smooth function. The **plane tangent** to a surface $z = f(x, y)$ at the point $P_0\,(x_0, y_0, z_0)$ is

$$z - z_0 = f_x\,(x_0, y_0)\,(x - x_0) + f_y\,(x_0, y_0)\,(y - y_0)\,,$$

where $z_0 = f\,(x_0, y_0)$.

Example1

1. Find the tangent plane to the surface $z = x^2 + 2y$ at the point $(1, 1, 3)$.
2. Find the plane tangent to the surface $z = x\cos y - y\,e^x$ at the point $(0, 0, 0)$

- **Linear Approximations**

1. The linearization of $f(x,y)$ at (x_0, y_0) is

$$L(x,y) = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0).$$

2. $L(x,y)$ is called the linear approximation of f at (x_0, y_0).

Example2:

1. Find the linearization of

$$f(x,y) = 2xe^{xy} \text{ at the point } (-1,0).$$

2. Use it to approximate $f(-1.1, 0.1)$.
3. Compare the approximation with the actual value of $f(-1.1, 0.1)$.