14 Partial Derivatives

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Outline of Chapter 14

1. Functions of Several Variables
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4. Tangent Planes and Linear Approximations
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6. Directional Derivatives and the Gradient Vector
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Definition

A function with more than two variables is a rule that assigns to each ordered pair $(x_1, x_2, \cdots, x_n)$ in a set D a unique real number denoted by $f(x_1, x_2, \cdots, x_n)$. D is called the domain of f and its range is $\{f(x_1, x_2, \cdots, x_n) \mid (x_1, x_2, \cdots, x_n) \in D\}$.

Example1

For each of the following functions with two variables, evaluate $f(2,1)$ and find the domain.

(1)

$$f(x,y) = \frac{\sqrt{x+y-1}}{x^2+y^2+1}$$

(2)

$$f(x,y) = y \ln(e^x + y^2)$$

Definition

If f is a function with more than two variables with domain D, then the graph of f is the set of all points $(x_1, x_2, \cdots, x_n) \in \mathbb{R}^n$ such that $x_n = f(x_1, x_2, \cdots, x_{n-1})$ and $(x_1, x_2, \cdots, x_{n-1}) \in D$.

Example2

The surface of the half sphere $g(x, y) = \sqrt{1 - x^2 - y^2}$ is a good example of graph.

Definition

The level curves of a function of more than two variables are the curves with equations $f(x_1, x_2, \cdots, x_n) = k$, where k is a constant.

Example3

Sketch the contour map of the function $g(x, y) = \sqrt{1 - x^2 - y^2}$ for $k = 0, 3/4, 1$.