

## 14.4 Tangent Planes and Linear Approximations

- Suppose that  $f$  is a smooth function. The **plane tangent** to a surface  $z = f(x, y)$  at the point  $P_0(x_0, y_0, z_0)$  is

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0),$$

where  $z_0 = f(x_0, y_0)$ .

### Example 1

1. Find the tangent plane to the surface  $z = x^2 + 2y$  at the point  $(1, 1, 3)$ .
2. Find the plane tangent to the surface  $z = x \cos y - y e^x$  at the point  $(0, 0, 0)$

- **Linear Approximations**

- 1 The linearization of  $f(x, y)$  at  $(x_0, y_0)$  is

$$L(x, y) = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0).$$

- 2  $L(x, y)$  is called the linear approximation of  $f$  at  $(x_0, y_0)$ .

### Example2:

1. Find the linearization of

$$f(x, y) = 2xe^{xy} \text{ at the point } (-1, 0).$$

2. Use it to approximate  $f(-1.1, 0.1)$ .
3. Compare the approximation with the actual value of  $f(-1.1, 0.1)$ .