- The method of Lagrange Multipliers provides a powerful tool for finding extreme values of **constrained functions.**
- In Example2 of the previous section, we found the shortest distance from the origin to the plane $x + 2y + z = 1$ **without** using Lagrange Multipliers.
- We need to figure out the concept of Lagrange Multipliers, drawing pictures.

Theorem

The method of Lagrange Multipliers

Suppose that the object function $f(x, y, z)$ and the constraint $g(x, y, z)$ are smooth and $\nabla g \neq \mathbf{0}$. To find the maximum and minimum values of f subject to $g(x, y, z) = k$,

1. Find the values of x, y, z and λ that simultaneously satisfy two equations

$$\nabla f = \lambda \nabla g \quad \textit{and} \quad g(x, y, z) = k.$$

2. Evaluate f at all the points (x, y, z) that result from step 1.

Example:

1. A rectangle box without a lid is to be made from $24m^2$ of cardboard. Find the maximum volume of such a box.
2. Find the greatest value and the smallest values that the function

$$f(x,y) = xy$$

takes on the ellipse

$$\frac{x^2}{4} + \frac{y^2}{2} = 1.$$

3. Find the maximum and minimum of the function $f(x,y) = 2x - 3y$ on the unit circle $x^2 + y^2 = 1$.
4. Find the maximum and minimum of the function $f(x,y) = x^2 + y^2$ on the curve $xy = 2$.
5. Find the maximum and minimum of the function $f(x,y) = 2x + y$ on the disk $x^2 + y^2 \leq 1$.