# 15.3 Double integrals in Polar Coordinates

- If the region $R$ is inside of a circle or a ring, a convenient way to evaluate double integrals is to use polar coordinates.
- $(x, y) \Rightarrow (r, \theta)$, where

$$r^2 = x^2 + y^2, \quad x = r\cos\theta, \quad y = r\sin\theta.$$

- Example: Consider the region $D = \left\{(x, y) \mid x^2 + y^2 \leq 1\right\}$. This can be switched to a polar rectangle:

$$R = \{(r, \theta) \mid 0 \leq r \leq 1, 0 \leq \theta \leq 2\pi\}.$$

## Change to Polar Coordinates in a Double Integral:

If $f$ is continuous on a polar rectangle $R$ given by $0 \leq a \leq r \leq b$, $\alpha \leq \theta \leq \beta$ , where $0 \leq \beta - \alpha \leq 2\pi$, then

$$\int\int_R f(x, y)\, dA = \int_{\alpha}^{\beta} \int_a^b f(r\cos\theta, r\sin\theta)\, r\, dr d\theta.$$