# 2. First Order DEs

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# Outline of Chapter 2

1. Linear Equations
2. Separable Equations
3. Existence and Uniqueness of Solutions of Nonlinear Equations
4. Transformation of Nonlinear Equations into Separable Equations
5. Exact Equations
6. Integration Factors

# Goal for Chapter 2

In this chapter, we study first order DEs which are written in the form:

$$\frac{dy}{dt} = f(t, y(t)), \tag{1}$$

where $f$ is a given function of $t$ and $y$.

1. Determine whether solutions $y = y(t)$ exist.
2. Develop methods for finding solutions. Note that for an arbitrary function $f$, there is no general method for solving the DE (1).
3. Several methods will be explained.

# 2.1 Linear Equations

- **First order linear DEs:** (1) is called a first order DE if the function $f$ is linearly dependent of the variable $y$. A typical example is $y' = \alpha y + \beta$, where $\alpha$ and $\beta$ are given functions.
- The **general** first order linear DEs have the standard form

$$\frac{dy}{dt} + p(t)y = g(t), \tag{2}$$

where $p$ and $g$ are given functions dependent of $t$. Unfortunately, it is not easy to solve the general equation (2). Leibniz provided a great idea to solve it by multiplying by a certain function $\mu(t)$ which is called an **integrating factor**.

- This idea allows us to set up the following formula

$$y(t) = e^{-\int p(t)dt} \int e^{\int p(t)dt} g(t)\, dt,$$

where $\int p(t)dt$ **is only the antiderivative of** $p$ **and** $\int e^{\int p(t)dt} g(t)dt$ **is the general antiderivative.**

- **If we cannot find the antiderivative of** $e^{\int p(t)dt} g(t)$, the solution of the equation (2) is

$$y(t) = e^{-\int p(t)dt} \left[ \int_{t_0}^{t} e^{\int p(s)ds} g(s)\, ds + C \right],$$

where $C$ is any constant.

## Example

Find solutions for the following first order DE

$$\frac{dy}{dt} = -3y + 1.$$

## Examples

**1. Solve the following first order DEs.**

(1) $y' + 2y = e^t$. (2) $y' - y = 3 + t$.

**2. Solve the initial value problem (IVP).**

$$(1) \begin{cases} ty' + y = 2t^2, \\ y(1) = 1. \end{cases} \qquad (2) \begin{cases} 2y' + ty, = 1, \\ y(0) = 1. \end{cases}$$