2.2 Separable Equations

- In this section, we consider first order DEs that can be solved by **direct integration**. Recall the general first order DE

$$\frac{dy}{dx} = f(x, y(x)). \tag{1}$$

By setting $M(x, y) = -f(x, y)$ and $N(x, y) = 1$, (1) becomes

$$M(x, y) + N(x, y)\frac{dy}{dx} = 0. \tag{2}$$

If M is a function in terms of only x and N is a function in terms of only y, the Eq. (2) can be written in the differential form

$$M(x)\,dx + N(y)\,dy = 0, \tag{3}$$

which is called to be **separable**. You are led to obtain an alternative differential form:

$$\frac{dy}{dx} = -\frac{M(x)}{N(y)}.$$

We use cross multiplication and take integrals to find solutions.

Examples

. Show that the following DE

$$\frac{dy}{dx} = \frac{x}{1+y^2}$$

is separable and find a solution for the DE.

2. Solve the initial value problem (**IVP**)

$$\frac{dy}{dx} = \frac{3x^2+2x+1}{2y-1}, \quad y(0) = 1$$

and find a solution explicitly.

3. Solve the IVP

$$y' = y^2 + 2xy^2, \quad y(0) = 1$$

and determine where the solution attains its maximum value.