2.6 Integrating Factors

Theorem

Let the functions $M, N, M_y,$ *and,* $N_x \in C((\alpha, \beta) \times (\gamma, \delta))$. *Then, the equation* $M(x,y) + N(x,y)y' = 0$ *is an* ***exact*** *DE in an open rectangular region* $(\alpha, \beta) \times (\gamma, \delta) \iff M_y(x,y) = N_x(x,y)$ *at each point of the rectangular region. That is,* $\exists$ *a function F such that* $F_x(x,y) = M(x,y), \quad F_y(x,y) = N(x,y) \iff M$ *and* N *satisfy the equation* $M_y(x,y) = N_x(x,y)$.

- If a DE is **not** exact, we find an **integrating factor** μ. See the following Theorem (pp. 84)
- We multiply by the integrating factor to get to an exact DE. Then we will use the same way as we did in the previous section.

Theorem

Let the functions $M, N, M_y,$ *and,* $N_x \in C((\alpha, \beta) \times (\gamma, \delta))$.
(a) If $(M_y - N_x)/N$ *is independent of* y *on* $(\alpha, \beta) \times (\gamma, \delta)$ *and we define*

$$p(x) = \frac{M_y - N_x}{N},$$

then an integrating factor will be

$$\mu(x) = e^{\int p(x)dx}.$$

(b) If $(N_x - M_y)/M$ *is independent of* x *on* $(\alpha, \beta) \times (\gamma, \delta)$ *and we define*

$$q(y) = \frac{N_x - M_y}{M},$$

then an integrating factor will be

$$\mu(y) = e^{\int q(y)dy}.$$