5.2 Constant Coefficient Homogeneous Equations

- All the steps to solve the homogeneous DEs with constant coefficients: $ay''(x) + by'(x) + cy(x) = 0$. See Theorem 5.2.1 in the textbook (p.217).

1. Switch it into the corresponding characteristic equation: $ar^2 + br + c = 0$.
2. Find its determinant $D = b^2 - 4ac$.
   (1) $D > 0 \Rightarrow r = r_1, r_2$, i.e, there are two distinct **real** roots. Then the solution will be $y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$
   (2) $D = 0 \Rightarrow r = r_1$, i.e, there is a repeated **real** root. Then the solution will be $y = e^{r_1 x}(c_1 + c_2 x)$.
   (3) $D < 0 \Rightarrow r = \lambda \pm i\,\omega$, i.e, there are two distinct **complex** roots. Then the solution will be
   $y = e^{\lambda x}(c_1 \cos \omega x + c_2 \sin \omega x)$.
3. The arbitrary values $c_1, c_2 \in \mathbb{R}$ will be determined when we solve IVPs.