

5.7 Variation of Parameters

- Variation of parameters supplements the method of undetermined coefficients and it can be applied to any second order nonhomogeneous DEs. So it is the general method.
- The method of variation of parameters requires us to evaluate certain integrals involving the nonhomogeneous term $g(x)$.
- See the Theorem in the next page.

Theorem

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Suppose that $p, q, g \in C(I)$ and $\{y_1, y_2\}$ is a fundamental set of solutions of the homo. DE corresponding to the nonhomo. DE

$$y'' + p(x)y' + q(x)y = g(x). \quad (1)$$

Then a particular solution Y of (1) is

$$Y(x) = -y_1(x) \int_{x_0}^x \frac{y_2(s)g(s)}{W(y_1, y_2)(s)} ds + y_2(x) \int_{x_0}^x \frac{y_1(s)g(s)}{W(y_1, y_2)(s)} ds, \quad (2)$$

where t_0 is any convenient point in the open interval I . Thus, the general solution is $y = c_1y_1(x) + c_2y_2(x) + Y(x)$.

- Usually, we find only antiderivatives in (2).
- Note that if a term in (2) belongs the fundamental set of solutions of Eq. (1), it will **not** be included in a particular solution.

Examples

- The undetermined coefficient method cannot be applied to the following examples.

Examples

1. Find a **particular** solution of $y'' - 3y' - 4y = 2e^{-x}$.
2. Find a **particular** solution of

$$y'' + y = \csc x.$$

3. Find the general solution of the following DE

$$y'' + 4y' + 4y = x^{-2}e^{-2x}.$$