7.2 Series Solutions Near an Ordinary Point, Part I

- Many physics applications give a rise to the following

$$P_0(x)y'' + P_1(x)y' + P_2(x)y = 0. \qquad (1)$$

$P_0(x)$, $P_1(x)$, $P_2(x)$ are polynomials in a wide class of problems in mathematical physics. For example,

1. **Bessel eq.**: $x^2y'' + xy' + (x^2 - \nu^2)\,y = 0$ for the constant ν.
2. **Legendre eq.**: $(1 - x^2)\,y'' - 2x\,y' + \alpha\,(\alpha + 1)\,y = 0$ for the constant α.

- Assume that P_0, P_1 and P_2 have no common factors. Then x_0 is called an **ordinary point** of (1), if $P_0\,(x_0) \neq 0$ Otherwise, then x_0 is called a singular point.

Theorem

(7.2.1) Assume that $P_0(x) \neq 0$, P_1 *and* P_2 *have no common factors. Let* x_0 *be a point such that* $P_0(x_0) \neq 0$. *Then every solution of*

$$P_0(x)y'' + P_1(x)y' + P_2(x)y = 0. \tag{2}$$

can be represented by a power series

$$y(x) = \sum_{n=0}^{\infty} a_n (x - x_0)^n$$

that converges at least on the open interval $(x_0 - R, x_0 + R)$.

Examples

- Set up a recursive relation to find power seres solution $y(x) = \sum_{n=0}^{\infty} a_n x^n$ about the center $x_0 = 0$.

1. $y'' - y = 0.$
2. $y'' + y = 0.$
3. $y'' - y' = 0.$
4. $y'' + y' = 0.$

- Seek power seres solution $y(x) = \sum_{n=0}^{\infty} a_n x^n$ about the center $x_0 = 0$:

$$y'' - xy = 0, \quad y(0) = y'(0) = 1.$$

Set up a recursice relation. Find a_n for $0 \le n \le 5$.