Theorem

Suppose that f, f' are continuous on $[0,\infty)$ and of exponential order s_0 and f' or f'' are piecewise continuous on $[0,\infty)$. Then f, f' and f' have Laplace transforms for $s > s_0$ and

$$\mathscr{L}\left(f'\right) = s\mathscr{L}(f) - f(0)$$

and

$$\mathscr{L}\left(f''\right) = s^2\mathscr{L}(f) - sf(0) - f'(0).$$

In general, $\mathscr{L}\left\{f^{(n)}(t)\right\}$ for $n \geq 1$ is given by

$$\mathscr{L}\left\{f^{(n)}(t)\right\} = s^n\mathscr{L}\left\{f(t)\right\} - s^{n-1}f(0) - \cdots - s\,f^{(n-2)}(0) - f^{(n-1)}(0).$$

- In order to solve IVPs with constants, we can use Laplace Transforms. Here are steps:

1. Use the Laplace Transform to transform an IVP for an unknown function f in the $t-$domain into **a simpler problem for F (or a nicer function F)** in the $s-$domain. The Theorem in the previous page will be applied.
2. Solve this algebraic problem to find F.
3. Invert F to obtain f.

Examples

Use the Laplace transforms to solve the following IVPs:

1. $y'' + y = 0, \quad y(0) = y'(0) = 1$
2. $y'' - y' - 6y = 0, \quad y(0) = 1, \quad y'(0) = -1$
3. $y'' - 2y' + 2y = 0, \quad y(0) = 0, \quad y'(0) = 1$
4. $y'' - 4y' + 4y = 0, \quad y(0) = y'(0) = 1$
5. $y'' + y = t, \quad y(0) = y'(0) = 1$