- **Laplace Transform of Piecewise Continuous Functions**
  1. In order to deal with jumping discontinuous functions, it will be convenient to introduce the **Heaviside function (unit step function),** denoted by u, which is defined by

$$u(t) = \begin{cases} 0, & \text{if } t < 0, \\ 1, & \text{if } t \geq 0. \end{cases}$$

  2. We can shift the unit step function by τ:

$$u(t-\tau) = \begin{cases} 0, & \text{if } t < \tau, \\ 1, & \text{if } t \geq \tau. \end{cases}$$

- **How to switch piecewise continuous functions to the linear combination of the (shifted) step unit functions $u(t-\tau)$.**

1. Assume that a function f consists of two pieces and is jump continuous at $t = t_1$, i.e.,

$$f(t) = \begin{cases} f_0(t), & \text{if } 0 \leq t < t_1, \\ f_1(t), & \text{if } t \geq t_1. \end{cases}$$

Then the piecewise continuous function f can be written by

$$f(t) = f_0(t) + u(t - t_1)(f_1(t) - f_0(t)).$$

2. Similarly, we cam write

$$f(t) = \begin{cases} f_0(t), & \text{if } 0 \leq t < t_1, \\ f_1(t), & \text{if } t_1 \leq t < t_2, \\ f_2(t), & \text{if } t \geq t_2. \end{cases}$$

as

$$f(t) = f_0(t) + u(t - t_1)(f_1(t) - f_0(t)) + u(t - t_2)(f_2(t) - f_1(t)).$$

Theorem

Suppose that $\tau \geq 0$ *and* $\mathscr{L}(g(t+\tau))$ *exists for* $s > s_0$.
(1) ***8.4.1*** *(pp. 423) Then* $\mathscr{L}(u(t-\tau)g(t))$ *exists for* $s > s_0$ *and*

$$\mathscr{L}\{u(t-\tau)g(t)\} = e^{-\tau s}\mathscr{L}\{g(t+\tau)\}.$$

(2) ***8.4.2*** *(pp. 426) (**Second Shifting Theorem**)*
$\mathscr{L}(u(t-\tau)g(t-\tau))$ *exists for* $s > s_0$ *and*

$$\mathscr{L}\{u(t-\tau)g(t-\tau)\} = e^{-\tau s}\mathscr{L}\{g(t)\} = e^{-\tau s}G(s),$$

where $\mathscr{L}(g(t)) = G(s)$.