1.3 Propositional Equivalences

Definitions

1. **A Tautology**: a compound proposition (c. p.) that is always **true**
2. **A Contradiction**: a c. p. that is always **false**
3. **A Contingency**: a c. p. that is neither a tautology nor a contra.

Example1

1. $p \vee \sim p$: tautology
2. $p \wedge \sim p$: contradiction

Hint: We can use the truth tables to check them out.

Definitions

1. Compound proposition (C. P.) that has the same truth values in all possible cases is called **logically equivalent (L.E.).**
2. The C. P.s p and q are called **L.E.** if $p \leftrightarrow q$ is a tautology. The notation $p \equiv q$ (or $p \Leftrightarrow q$) denotes that p and q are called L.E..

Several Laws

- **De Morgan's laws:**
  1. $\sim(p \wedge q) \equiv \sim p \vee \sim q$ **2.** $\sim(p \vee q) \equiv \sim p \wedge \sim q$
- **Two distributive laws:**
  1. $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
  2. $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Example2

1. Prove De Morgan's laws.
2. Prove the two distributive laws.
3. Show that $p \to q \equiv \sim p \bigvee q$.

- **Conditional disjunction equivalence (Look at p.28) which is a significant role in understanding all the equivalences:** $p \to q \equiv \sim p \vee q$

- **Some Important L.E.s**

Equivalence	Name	Equivalence	Name
$p \wedge T \equiv p$ $p \vee F \equiv p$	Identity	$(p \bigvee q) \bigvee r \equiv p \bigvee (q \bigvee r)$ $(p \bigwedge q) \bigwedge r \equiv p \bigwedge (q \bigwedge r)$	Assoc.
$p \vee T \equiv T$ $p \wedge F \equiv F$	Domi.	$p \bigwedge (q \bigvee r) \equiv (p \bigwedge q) \bigvee (p \bigwedge r)$ $p \bigvee (q \bigwedge r) \equiv (p \bigvee q) \bigwedge (p \bigvee r)$	Distri.
$p \wedge p \equiv p$ $p \vee p \equiv p$	Idemp.	$\sim (p \bigwedge q) \equiv \sim p \bigvee \sim q$ $\sim (p \bigvee q) \equiv \sim p \bigwedge \sim q$	De Mor.
$\sim (\sim p) \equiv p$	D.neg.	$p \bigvee (p \bigwedge q) \equiv p$ $p \bigwedge (p \bigvee q) \equiv p$	Absorp.
$p \wedge q \equiv q \wedge p$ $p \vee q \equiv q \vee p$	Comm.	$p \bigwedge \sim p \equiv F$ $p \bigvee \sim p \equiv T$	Nega.

L.E.s involving Conditional Statements

Equivalence	Equivalence
$p \to q \equiv \sim p \vee q$	$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$
$p \to q \equiv \sim q \to \sim p$	$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$
$p \vee q \equiv \sim p \to q$	$(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$
$p \wedge q \equiv \sim (p \to \sim q)$	$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$
$\sim (p \to q) \equiv p \wedge \sim q$	

Example3

1. Show that $\sim (p \to q) \equiv p \wedge \sim q$.
2. Show that $\sim (p \vee (\sim p \wedge q)) \equiv \sim p \wedge \sim q$.
3. Show that $(p \wedge q) \to (p \vee q) \equiv \mathrm{T}$.