

## 1.6 Rules of Inference

- **Inference:** an act or the process of concluding or deciding from something known or assumed.
- **An Argument:** a sequence of statements (propositions) which end with a conclusion.
- An argument is **valid** if and only if it is impossible to have the compound proposition such that all premises are true and the conclusion is false. In another word, an argument is **invalid** if and only if it we do have the compound proposition such that all premises are true and the conclusion is false. **Therefore, an argument is valid  $\Leftrightarrow$  it is a tautology.**

### Definitions

1. An argument in propositional logic is a sequence of propositions. All but the final proposition in the argument are called premises and the final proposition is called the conclusion.
2. An argument is **valid** if the truth of all its premises implies that the conclusion is true.

# Valid Arguments in Propositional Logic

- From the definitions, an argument form (using a logical expression) will be used to apply rules of inference.
- Consider the following argument:

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“If you are an ASU student, then you can log onto Mycampus.”

“You are an ASU student.”

Therefore,

“You can log onto Mycampus.”

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Then we will use a logical expression to determine its validity.

Let  $p$  : you are an ASU student and  $q$  : you can log onto Mycampus. Then, the argument has the form

$$p \rightarrow q$$

$$p$$

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$$\therefore q$$

# Rules of Inference

Rules of Inference	Tautology	Name
$\begin{array}{l} p \\ p \rightarrow q \\ \hline \therefore q \end{array}$	$(p \wedge (p \rightarrow q)) \rightarrow q$	Modus ponens
$\begin{array}{l} \sim q \\ p \rightarrow q \\ \hline \therefore \sim p \end{array}$	$(\sim q \wedge (p \rightarrow q)) \rightarrow \sim p$	Modus tollens
$\begin{array}{l} p \rightarrow q \\ q \rightarrow r \\ \hline \therefore p \rightarrow r \end{array}$	$(p \rightarrow q) \wedge (q \rightarrow r) \\ \rightarrow (p \rightarrow r)$	Hypothetical syllogism
$\begin{array}{l} p \vee q \\ \sim p \\ \hline \therefore q \end{array}$	$((p \vee q) \vee \sim p) \rightarrow q$	Disjunctive syllogism

# Rules of Inference

Rules of Inference	Tautology	Name
$\begin{array}{l} p \\ \text{-----} \\ \therefore p \vee q \end{array}$	$p \rightarrow (p \vee q)$	Addition
$\begin{array}{l} p \wedge q \\ \text{-----} \\ \therefore p \end{array}$	$(p \wedge q) \rightarrow p$	Simplification
$\begin{array}{l} p \\ q \\ \text{-----} \\ \therefore p \wedge q \end{array}$	$(p \wedge q) \rightarrow p \wedge q$	Conjunction
$\begin{array}{l} p \vee q \\ \sim p \vee r \\ \text{-----} \\ \therefore q \vee r \end{array}$	$((p \vee q) \wedge (\sim p \vee r)) \rightarrow (q \vee r)$	Resolution

# Rules of Inference for Propositional Logic

- Consider another argument form, called modus ponens:

$$\begin{array}{l} p \\ p \rightarrow q \\ \hline \therefore q \end{array}$$

The hypotheses are written in a column, followed by horizontal dashed (or solid) line that begins with the therefore symbols and ends with the conclusion.

## Example 1

- State which rule of inference is the basis of the following argument: “It is below freezing now. Therefore, it is either below freezing or raining.”
- State which rule of inference is used in the following argument: “It is below freezing and raining now. Therefore, it is below freezing now.”

- **Fallacies** (incorrect reasoning) lead to invalid arguments.
- There are two types of Fallacies.
  1.  $((p \rightarrow q) \wedge q) \rightarrow p$  is not tautology. This is called the **fallacy of affirming the conclusion**.
  2.  $((p \rightarrow q) \wedge \sim p) \rightarrow \sim q$  is not tautology. This is called the **fallacy of denying the hypothesis**.

## Example 2

1. Is the following argument valid?

“If you do every problem in Calculus book, then you will learn discrete mathematics. You learned discrete mathematics.

Therefore, you did every problem in Calculus book.

2. For the following arguments determine whether the argument is correct or incorrect and explain why.

“ All parrots like fruit. My pet bird is not a parrot. Therefore, my pet bird does not like fruit.”

- If there are many premises, several rules of inference are needed to show that an argument is valid. However, we will not consider the case.

### Example 3

1. What rule of inference is used in the following arguments? “If it snows today, ASU will close. ASU is not closed today. Therefore it did not snow today.”
2. What rules of inference are used in this famous argument? “All men are mortal. Socrates is a man. Therefore, Socrates is mortal.”
3. For the following collection of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain the conclusion from the premises. “I am either clever or lucky.” “I am not lucky.” “If I am lucky, then I will win the lottery.”
4. Show that the argument form with premises  $p_1, p_2, \dots, p_n$  and conclusion  $q \rightarrow r$  is valid if the argument form with the premises  $p_1, p_2, \dots, p_n$  and conclusion  $r$  is valid.