2. Basic Structures: Sets, Functions, Sequences, Sums, and Matrices

Department of Mathematics & Statistics

ASU

Outline of Chapter 2

1. **Sets**
2. **Set Operations**
3. **Functions**
4. **Sequences and Summations**
5. Cardinality of Sets
6. **Matrices**

Definition

A Set is a well-defined collection of distinct objects, called elements or members of the set. **Well-defined** means that there is a clear rule that enables us to determine whether a given object is an element of the set.

- **Notations**
  If a belongs to A, we write $a \in A$. If not, we write $a \notin A$. Usually, sets are denoted by uppercase letters such as $A, B, \cdots$ and elements are denoted by lowercase letters such as $a, b, \cdots$.
- **Three ways to describe a set**
  1. List(Roaster) method: all elements are listed between braces. **Ex:** $A = \{a, b, c\}$
  2. Set builder notation: by stating the property that elements must satisfy. **Ex:** $\mathbb{R}^+ := \{x \in \mathbb{R} \mid x \geq 0\}$
  3. Interval notation, especially for real numbers. Ex: $[a, b]$ or $[a, b)$

Definition

$A = B$ if two set A and B have the same elements regardless their order.

Example1

1. $\{1,3,5\} = \{1,5,3\} = \{1,1,3,5,5\}$

- **Venn Diagram**
  is convenient to understand relations between sets.The universal set U is represented by a rectangle. Normally circles are used to represent certain sets and points are used to represent the particular elements of the set.

Example2

Use to a Venn Diagram to show that if $A \subset B$ and $B \subset C$, then $A \subset C$.

- Notations for the empty (null) set: $\emptyset$ or $\{\}$.
  Don't be confuse $\{\}$ and $\{\emptyset\}$. Those are not the same!

Definition

The set A is a subset ($A \subseteq B$) if every element of A is also an element of B.

- We can use the quantification to define a subset:

$$\forall x(x \in A \rightarrow x \in B)$$

Theorem

For every set S, (1) $\{\} \subseteq S$ (2) $S \subseteq S$.

- When we want to emphasize that A is a subset of B but $A \neq B$, we write $A \subset B$ and say that A is a proper subset of B. We can also use the quantifications to define a subset:

$$\forall x(x \in A \rightarrow x \in B) \wedge \exists x(x \in B \wedge x \notin A)$$

is true.

Definitions

1. The **cardinality of** a set S means the number of elements in the set S. It is denoted by $|S|$ and $|S| = n \geq 0$ for all integers.
2. If n is the finite number, then S is called a finite set. If S is not finite or $(n = \infty)$, then S is called an infinite set.

Example3

1. Let S be the set of English alphabet letters. |S|=26.
2. $|\emptyset| = 0$ 3. $S = \{s \in \mathbb{N} \mid 0 \leq s \leq 5\}$ Then $|S| = 5$
4. $|\mathbb{Z}| = \infty$.

Definitions

1. For a given set S, the power set of the set S is the set including all subsets of the set. The power set of S is denoted by $\mathscr{P}(S)$.
2. If $|S| = n$, then $|\mathscr{P}(S)| = 2^n$.

Example4

1. Let $S = \{0, 1, 2\}$. Then find $\mathscr{P}(S)$ and $|\mathscr{P}(S)|$.

Definitions

1. The **ordered** $n-$**tuples** $(a_1, a_2, \cdots, a_n)$ is the ordered collection that has a_1 as its first element, a_2 as its second element, ..., and a_n as its nth element. In particular, ordered 2−tuples are called ordered pairs.
2. The **Cartesian product** of A and B, denoted by $A \times B$, is the set of all ordered pair (a, b) such that $a \in A$ **and** $b \in B$. Thus

$$A \times B = \{(a, b) \mid a \in A \wedge b \in B\}.$$

Example5

1. $A = \{0, 1\}$ and $B = \{x, y, z\}$. Then find $A \times B$.
2. Show that $A \times B \neq B \times A$.