2.2 Set Operations

- You may recall algebra operations and logic operations, before considering set operations.

Definitions

1. The **union** of sets A and B, denoted by $A \cup B$, is the set that contains those elements of A **or** B **or both.** Thus, we can say that $A \cup B = \{x \mid x \in A \vee x \in B\}$.
2. The **intersection** of sets A and B, denoted by $A \cap B$, is the set that contains those element in both A **and** B**.** Thus, we can say that $A \cap B = \{x \mid x \in A \wedge x \in B\}$.
3. The sets A and B are called disjoint if $A \cap B = \emptyset$.
4. The difference of A and B, denoted by $A - B$ (or $A \backslash B$) is defined by $A - B = \{x \mid x \in A \wedge x \notin B\}$.
5. Let U be the universal (whole) set. The complement of the set A, denoted by $\overline{A}$ is defined by $\overline{A} = \{x \in U \mid x \notin A\}$. Thus, we can say that $\overline{A} = U - A$.

Example1

1. Let U be the set of whole number, i.e., $U = \mathbb{N} \cup \{0\}$. Let $A = \{1,2,3,4,5\}$ and $B = \{0,2,4\}$. Then find
(1) $A \cup B$ (2) $A \cap B$ (3) $A - B$ (4) $B - A$.
2. Show that $A - B = A \cap \overline{B}$.

- **Computer Representation of Sets**
  Assume that $U = \{a_1, a_2, \cdots, a_n\}$ is finite. We represent $A \subset U$ with the bit string of length n, where the ith bit with $1 \le i \le n$ in this string is 1 if $a_i \in A$ and is 0 if $a_i \notin A$.

Example2

Let $U = \{n \in \mathbb{N} \mid n \le 10\}$.
1. Express each of these sets with bit strings where the ith bit in the string is 1 if i is in the set and 0 otherwise.
(1) $\{3,4,5\}$ (2) $\{1,3,6,10\}$
2. Find the set specified by each of these bit strings.
(1) 11 1100 1111 (2) 10 0000 0001.

Set Identities

Identities	Name	Identities	Name
$A \cap U = A$ $A \cup \emptyset = A$	Identity laws	$A \cup U = U$ $A \cap \emptyset = \emptyset$	Domination laws
$A \cap A = A$ $A \cup A = A$	Idempotent	$\overline{\overline{A}} = A$	Complementation
$A \cap B = B \cap A$ $A \cup B = B \cup A$	Commutative	$\overline{A \cap B} = \overline{A} \cup \overline{B}$ $\overline{A \cup B} = \overline{A} \cap \overline{B}$	De Morgan's
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption	$A \cap \overline{A} = \emptyset$ $A \cup \overline{A} = U$	Complement

Identities	Name
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative laws
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distribution laws

- **We can check all the rules, using Venn diagrams.**