- When we express integers, we usually use decimal notation. For example,

$$742 = 7 \times 10^2 + 4 \times 10 + 2.$$

However, it is convenient to use bases other than 10. We introduce the following theorem which is the most general case and then we will consider binary, octal, and hexadecimal expansions.

Theorem

Let $b > 1$ be an integer. Then if n is a positive integer, it can be expressed uniquely in the form

$$n = a_k b^k + a_{k-1} b^{k-1} + \cdots + a_1 b + a_0,$$

where $k \geq 0$ is an integer, $a_0, a_1, a_2, \cdots, a_k$ are nonnegative integers less than b and $a_k \neq 0$. b is called a basis.

- **Binary Expansions**
  are given, when we choose 2 as the basis. So each digit is either 0 or 1, since they are only nonnegative numbers less than 2.
- **Octal Expansions**
  are given with the basis 8. Each digit is $0, 1, 2, 3, 4, 5, 6, 7$
- **Hexadecimal Expansions**
  are given with the basis 16.
  Each digit is $0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F$.

Example

1. Convert the following decimal expansion of integers to a binary expansion: (1) 231 (2) 97644
2. Convert the following binary expansion to a decimal expansion: (1) $(1\,1111)_2$ (2) $(1\,0101\,0101)_2$
3. Convert the following octal expansion to a binary expansion: (1) $(1604)_8$ (2) $(2417)_8$
4. Convert the following binary expansion to an octal expansion: (1) $(1111\,0111)_2$ (2) $(101\,0101\,0101\,0101)_2$
5. Convert the following hexadecimal expansion to a binary expansion: (1) $(80E)_{16}$ (2) $(ABBA)_{16}$
6. Convert $(1011\,0111\,1011)_2$ from its binary expansion to its hexadecimal expansion.