- Considering recursive definitions is one of a good way to find sequences. Recall that a sequence a_n is a particular function $f(n)$, where the domain is the set of nonnegative integers. Let $a_n = f(n)$.
- We use **two steps** to consider recursive definitions where the **next term** is expressed by the **previous terms**.

1. **Basis step:** Specify $f(0)$ or the first term a_0.
2. **Recursive step:** Provide a rule for finding the next term $a_{n+1} = f(n+1)$ from the previous term $a_n = f(n)$.

Example

1. Find $f(1)$, $f(2)$, $f(3)$, and $f(4)$ if $f(n)$ is defined recursively by $f(0) = 1$ and for all integers $n \geq 0$,

(1) $f(n+1) = f(n) + 2$

(2) $f(n+1) = 3f(n)$

(3) $f(n+1) = 2^{f(n)}$

(4) $f(n+1) = [f(n)]^2 + f(n) + 1$

2. Give a recursive definition of the sequence $\{a_n\}$, $n = 1, 2, 3, \cdots$

(1) $a_n = 6n$ (2) $a_n = 2n + 1$ (3) $a_n = 10^n$ (4) $a_n = 5$

3. Let F be the function such that $F(n)$ is the sum of the first positive integers. Give a recursive definition of $F(n)$

4. Give a recursive definition of $P_m(n)$, the product of the integer m and nonnegative integer n.

5. Give a recursive definition of

(1) the set of even numbers

(2) the set positive integers not divisible by 5.