6. Counting

Department of Mathematics & Statistics

ASU

Outline of Chapter 6

1. **The Basics of Counting**
2. **The Pigeonhole Principle**
3. **Permutation and Combinations**
4. **Binomial Coefficients and Identities**
5. Generalized Permutations and Combinations
6. Generating Permutations and Combinations

- Combinatorics, a branch of mathematics is to study finite or countable discrete structures. Counting is used to determine the complexity of algorithms. Recently, it has played a key role in applying mathematics to biology especially in sequencing DNA.

6.1 The Basics of Counting

- When we study applied mathematics or statistics or computer science, counting problems arise.
- There are two basic counting principles; the **product rule** and the **sum rule.**

1. The product rule can be considered when a procedure is made up of separate tasks.
   THE PRODUCT RULE: Suppose that a procedure can be broken into a sequence of two tasks. If there are n_1 ways to do the first task and for each way n_1 there are n_2 ways to do the second task, then there are $n_1 n_2$ ways to do the procedure.
2. Here is the second rule called the sum rule.
   THE SUM RULE: Suppose that a task can be done **either** in one of n_1 ways **or** in one of n_2 ways. If any of the set of $n_1$ways is **not** the same as any of the set of n_2 ways, then there are $n_1 + n_2$ ways to do the task.

- We think of two more counting rules, when we have more complicated counting problems.

1. **THE SUBTRACTION RULE**: If a task can be done in either n_1 ways or n_2 ways, then the total number of ways is $n_1 + n_2 - n_3$, where n_3 is the number of ways to do the task that care common to the two different ways.
   This rule is known as the **principle of inclusion-exclusion.**
   This can be understood in terms of considering sets:
   $$|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|.$$
2. The division rule can be useful, when we solve certain types of enumeration problems
   THE DIVISION RULE: There are n/d ways to do a task if it can be done using a procedure that can be carried out in n ways and for every way n exactly d of the n ways correspond to the way w.
   Similarly, suppose that $|A| = |A_1| + |A_2| + \cdots + |A_n|$, where each subset $|A_i| = d$ with $1 \le i \le n$. Then $n = |A|/d$.

Example

1. There are 20 mathematics majors and 30 computer science majors at ASU.
(1) In how may ways can two representatives be picked so that one is a mathematics major **and** the other is a computer science major?
(2) In how may ways can two representatives be picked so that one is a mathematics major **or** the other is a computer science major?
2. How many positive integers between 50 and 100.
(1) are divisible by 7?
(2) are divisible by 11?
(3) are divisible by 7 and 11?
3. How many strings of three decimal digits
(1) do not contain the same of digit three times?
(2) begin with an odd digit?
(3) have exactly two digits that are 4s?
4. How many license plates can be made using either two uppercase English letters followed by four digits or two digits followed by four uppercase English letters?