- **Error Formula for Trapezoidal rule**

Theorem

Assume that $f \in C^2[a,b]$ *and* $I(f) = \int_a^b f(x)\,dx$. *Then the error formula is*

$$E_n^T(f) = I(f) - T_n(f) = -\frac{h^2}{12}(b-a)\,f''(c_n)\,, \qquad (1)$$

where c_n *is in between* a *and* b *and* $h = (b-a)/n$. *We can also obtain*

$$\left|E_n^T(f)\right| = \frac{h^2}{12}(b-a)\left|f''(c_n)\right|. \qquad (2)$$

Example1

Check the error formula (2) with $I(f) = \int_0^1 1/(2x+1)\,dx$.

- **An Asymptotic Estimate of the Trapezoidal Error.**

$$E_n^T(f) \approx -\frac{h^2}{12}\left(f'(b) - f'(a)\right) =: \widetilde{E}_n^T(f), \qquad (3)$$

where n is the number of subintervals. We can also derive the following;

$$\left|E_n^T(f)\right| \approx \frac{h^2}{12}\left|f'(b) - f'(a)\right| =: \left|\widetilde{E}_n^T(f)\right| \qquad (4)$$

The asymptotic esitmate (3) is practically better than (1).

Example2

Check the asymptotic estimate (4) with $I(f) = \int_0^1 1/(2x+1)\,dx$.

- **Error Formula for Simpson rule**

Theorem

*Assume that $f \in C^4[a,b]$ and let n be an **even** positive integer. Then the error formula is*

$$E_n^S = I(f) - S_n(f) = -\frac{h^4(b-a)}{180} f^{(4)}(c_n) \text{ and } \left|E_n^S\right| = \frac{h^4(b-a)}{180} \left|f^{(4)}(c_n)\right|$$

*where c_n is in between a and b and $h = (b-a)/n$. The **asymptotic estimate** can be*

$$\widetilde{E}(f) \approx -\frac{h^4}{180}\left(f'''(b) - f'''(a)\right) =: \widetilde{E}_n^S(f) \text{ and}$$

$$\left|E_n^S(f)\right| \approx \frac{h^4}{180}\left|f'''(b) - f'''(a)\right| =: \left|\widetilde{E}_n^S(f)\right|.$$

Example3

Check the asymptotic estimate with $I(f) = \int_0^1 1/(2x+1)\,dx$.