6.2 Euler's methods

- Differential equations are used to model and understand systems(physical situations) mostly with time variable t.
- Recall IVP; for $t \in [a, b]$

$$\begin{cases} \frac{dy}{dt} = f(t, y(t)) \\ y(a) = y_a \end{cases}$$

- When f is not a simple function, numerical methods will be used to obtain approximations; the popular methods are **Euler's methods (backward or forward)**

The explicit Euler's Method(Forward)

1. We partition the interval $[a, b]$:

$$a = t_0 < t_1 < t_2 < \cdots < t_n = b,$$

where $h = t_n - t_{n-1}$ with $n \geq 1$ is the size of the subintervals.

2. Let w_n be the approximation at t_n for $n \geq 0$. Recalling the IVP, we can set up the iterative formula:

$$w_0 = y_0$$
$$w_{n+1} = w_n + h f(t_n, w_n).$$

Example1

1. Find the Euler's method formula for the following IVP: for $t \in [0, \infty)$

$$\begin{cases} \frac{dy}{dt} = 2y \\ y(0) = 1 \end{cases}$$

2. Show that the numerical solution by Euler's method converges to the exact solution.

Implicit Euler's Method(Backward)

1. Here is the iterative formula for the implicit Euler's method;

$$w_0 = y_0$$
$$w_{n+1} = w_n + hf(t_{n+1}, w_{n+1}).$$

2. When you use the implicit Euler method, it is not easy to solve for the next step solution w_n. But the implicit Euler method is stable numerically.

Example2

Do the same as you did in previous Example1.

Example3

Use both methods to get approximations for the IVP:

$$\begin{cases} \frac{dy}{dt} = -2ty, \\ y(0) = 1. \end{cases}$$