Measurement: An Introduction

Most people need to be familiar with taking measurements, as almost any occupation requires measurements of some kind to made. Carpenters measure boards for cutting, nurses measure the blood pressure of patients, tailors measure fabric for garments, and advertising executives measure the public's acceptance of their sales pitches. You will, therefore, undoubtedly be utilizing measurement in your chosen career, regardless of the field you enter.

Measurement plays a particularly large role in science. Scientists gather data as a part of their studies, and to do this, they measure things. Scientists measure the concentration of gases in the atmosphere, the growth of organisms under varying conditions, the rate of biochemical reactions, the distance of stars from Earth, and an innumerable number of other things. As measurements form the basis of scientific inquiry, they are deserving of in-depth analysis in lab.

In a scientific experiment, the investigator examines the effects of variations in the independent variable on the dependent variable through measurements. For example, let's assume a biologist is studying the effect of temperature on plant growth. She sets up several different temperature conditions, and grows groups of plants from seedlings in each condition. When the experiment ends, she must compare plant growth in the plants from different temperatures. But how should she do this? Should she just look at the plants and decide which grew the best? Should she pick up the plants and "feel" which ones have the greatest mass? Of course not. She would use some sort of quantitative measurement, such as measuring the height of each plant's stem in centimeters or determining the total plant biomass in grams. Whichever measurement she chooses, she would need to utilize an instrument to make it.

Instruments

Take a minute and look around you at the variety of objects in your environment. There are probably a few pens or pencils, a notebook or two, assorted computer equipment, and other things. While you may be able to easily distinguish the differences between some objects (e.g., your notebook is longer than your pen), other differences are more difficult to discern. You may not be able to easily determine, for example, whether your computer keyboard or your course textbook has greater mass. Even when we can distinguish differences, it is not always easy to determine the extent of those differences. You may have noted that your computer monitor is heavier than your notebook, but is it twice as heavy or three times as heavy? Using only our senses, we cannot be certain about the answer, so we must take measurements.

Fig. 1: Bronze yard No. 11 (*NIST*)

To take measurements, we need instruments that are calibrated to known standards. Instruments include simple things like rulers and graduated cylinders, and complicated electronics like pH sensors and mass spectrometers. All of these instruments provide us what our senses cannot - a quantified measure of the properties of an object. In this week's lab, you will get an introduction to measurement by using various instruments to determine useful quantities around the home. Before proceeding, though, a few short comments about units are necessary.

System of Units: Metric Versus British

If you are measuring something, you need "units" to describe the object. In formal terms, a scale of measurement is the assignment of numbers or symbols to measure an attribute. In the past, natural units of measurement, such as a "foot", were commonly used. Unfortunately, these units were somewhat arbitrary. In Roman times, for example, a "foot" in England was what today would be 29.6 centimeters (the centimeter did not exist back then). When the Saxons took over, the size of a "foot" grew to 33.5 cm. Five centuries later, it was reduced to 30.5 cm. Finally, in 1959, the "International Foot" was defined as 30.48 cm. Even today, a "foot" in England is different from a "fod" in Denmark (31.41 cm), a "fod" in Sweden (29.69 cm), and a "fuss" in Germany (31.61 cm). With the increase in international trade during the 18th century, merchants needed to standardize units of measurement. This resulted in the development and nearly universal adoption of the International System of Units (SI) or metric system around the world. Of course, the United States is a notable exception to this worldwide trend, as we continue to use the English system of measurement. We buy our gas in gallons, measure our weight in pounds, and gauge driving distances in miles. The metric system has crept into our society somewhat (e.g., the two-liter soda bottle), but universal acceptance of this system of measurement anytime soon is unlikely.

In science, use of the metric system is unquestioned. Because of its international familiarity and ease of use, scientific studies utilize metric measurements. However, as this course will involve measuring certain quantities about your everyday life, we will not necessarily measure everything in metric units. Sometimes, we will use the British system, while other times will involve metric. Therefore, you will need to be conversant in both. Since most of you have grown up with the British system, we will not review it here. Furthermore, as you have dealt with the SI (metric) system throughout your K-12 years, a review of the system will not be provided here, either. If you need a refresher, please visit the web sites below for additional information.

The NIST Reference on Constants, Units, and Uncertainty
Physics Laboratory at NIST
http://physics.nist.gov/cuu/Units/index.html

The Metric System
Gordon Speer
http://www.essex1.com/people/speer/metric.html

How Many? A Dictionary of Units of Measurement
Russ Rowlett, University of North Carolina
http://www.unc.edu/~rowlett/units/index.html

Accuracy and Resolution

One question you might have after taking a measurement is “Is this the correct measurement?” That is, how close is the answer you have to the real answer? In answering this question, you need to consider two terms that are often mistaken as being the same thing: accuracy and precision. Accuracy refers to how close your measurement is to the actual value that the object has. There are many factors that can affect the accuracy. Was your measuring device calibrated correctly? Did you read the value from the instrument correctly? Did you place measuring device in the right place to take the measurement? If any of these factors is wrong, then you know that the accuracy of your measurement will be affected.

However, even if you do everything correctly and have the best measuring device manufactured, your measurement will still not be absolutely accurate. The reason for this is because of the precision of your measuring device, which refers to how repeatable your measurement is. No device made can measure a quantity to infinite precision. For example, look at a thermometer (Figure 2). There is a

Fig. 2: Mercury thermometer *(NIST)*

line that shows you the temperature 100^{o}. But where on the thermometer is the line 100.1^{o} or 99.9^{o}? As you can see, there is none. Furthermore, the line that demarcates 100^{o} has a certain thickness to it. This means that different parts of the line correspond to different temperatures, such as 100.00000^{o} and 100.00001^{o}. Because of this, we say that this thermometer is only precise to the nearest degree.

This leads us to a potential problem. Some books will say that you should interpolate values on such a thermometer. For instance, if the mercury were to terminate somewhere between 99^{o} and 100^{o}, they will instruct you to estimate how far between the lines that it is and then quote are result such as 99.4^{o}. Unfortunately, this is leads to problems in making calculations later on and should be done with extreme care. Interpolating like this assumes that the instrument was already calibrated to infinite precision when the original values were painted onto the thermometer. This is not true. When the manufacturer created the thermometer, they knew that they could only be so precise in creating the cylindrical hole in the thermometer. Furthermore, they realized that the markings would have to have a certain thickness and would cover a range of degrees. They purposely created the instrument only to be valid to the nearest degree, not to the nearest tenth of a degree.

Interpolating the value between the marks erroneously assumes precision in an instrument that is just not there. If one does interpolate the value, they need to signify the amount of error in their data. For instance, with the measurement above, the value should be stated as $99.4^{o} \pm .5^{o}$, where the plus/minus sign denotes the amount of possible error in the measurement. This creates problems when you use this number in any calculation, as you have to also take into account how this much error will affect the final calculation. This problem can be eliminated by not interpolating and keeping the data to the correct number of significant digits.

Significant Digits and Scientific Notation

The precision of your measurements makes itself apparent in the number of digits that you should use in any calculation. For instance, if your ruler can measure length to the nearest .01 meters, then you should not express any length to more than 2 digits to the right of the decimal point when you use units of meter. This will limit the number of **significant digits** that you have, which is vitally important to know if you are going to use your measurements in calculations. Using the example ruler above, we might find that a particular brick has a width of 34.02 m, a length of 12.24 m, and a height of 23.63 m. All of these measurements have 4 significant digits. If we use them to calculate the volume of the rectangular brick, we will need to round off any result to 4 significant digits, as this is the lowest number of significant digits of any of the measurement. In other words, the volume would be reported as

$$\mathbf{(34.02\ m) \times (12.24\ m) \times (23.63\ m) = 9840\ m^3}$$

In the value for the volume, the last digit (0) is still significant. However, it might be confused for not being significant because it is 0, which might just be there as a placeholder, i.e. 9840 with only three significant digits looks very much like 9840 with four significant digits. One way to show that the last digit is significant is to use **scientific notation**, where numbers are expressed with one digit to the left of the decimal times the appropriate power of ten. Using this, the volume of the brick would be expressed as $9.840 \times 10^3\ m^3$. If the number had had only 3 significant digits, it would have been written without the last 0, or $9.84 \times 10^3\ m^3$.

One thing to keep in mind when using measurements in calculations is that you must round to the smallest numbers of significant digits of any of your numbers. For instance, if the length of the brick had been 9.73 m, then the volume would have been

$$\mathbf{(34.02\ m) \times (9.73\ m) \times (26.63\ m) = 8.81 \times 10^3\ m^3}$$

where we now only have 3 significant digits in the final answer. This might be one reason why some instructors insist on interpolating results on thermometers inappropriately. If you can only measure the temperature as 88 ^{o}C, then any calculation that you make with this number must be limited to 2 significant digits, which is fairly limiting.

Error Analysis

The purpose of any experiment is to test how well a particular theory mirrors reality. Since theories involve a simplification of nature in order to allow us a better understanding of it, we know that the results of any experiment that we do will not perfectly match those that our theory predicts. The question that we always ask at the end of the experiment is, "How close do our theoretical predictions come to reality?" To answer this, we have to know how well our experimental apparatus and procedure matches the assumptions of our theory. This means that we need to do error analysis to see what limits we must put on our experimental data.

The term "error analysis" is often confusing to students. Most think that the word "error" is being used to mean "goof up" and refers to something that was done "wrong" in the experiment. This is not the case. If you know that you "goofed up" the procedure for an experiment, you are supposed to go back and redo the experiment the correct way. Instead, the term "error" here is meant to refer to things in the experimental set up that are not taken into account by one's theory. An example is the fact that measuring devices cannot measure things to infinite precision. Because of this, there will be some inherent "error" between what you theoretically predict and what you measure.

There are two types of errors in any experiment: random and systematic. Random errors are those that will change value and sign every time that a measurement is taken. For example, a spring might be used to launch a projectile in some experiment. The amount of force that the spring delivers to the projectile will depend up a variety of factors such as temperature, initial compression, etc. If the experiment is run multiple times, there will be some randomness to these factors, which will result in a small random change in the amount of force delivered to the projectile. Because these changes are random, one can account for random errors by running experiments multiple times with the same parameters and averaging the result.

Systematic errors are those factors that will always bias the result one way or another. In the projectile example above, suppose that we have a theory about how the projectile will move that does not account for wind resistance. If we run the experiment with any atmosphere in the chamber, this air will act upon the projectile in some manner. Because the wind resistance will operate upon the projectile the same way each time, it will bias the final result in a particular direction (ex. the ball does not go as far because wind resistance was slowing it down). Since there is no way to average out this error, the only way to account for random errors is by analyzing your theory to see in which direction such an error would bias the result and try to make an estimate as to how great the bias will be.

Activity

The activity for this week will consist of two parts. In part one of the lab, we are going to practice measuring spatial dimensions with some common items and use these measurements to calculate some relevant quantity. In particular, we are going to measure the dimensions of food cans and use these to calculate the density of the can and contents. To do this part, you will need to get three different canned goods to measure. For instance, you might wish to try soup, corn, and tomato paste as the different canned products you measure. If you do not happen to have any of these cans around your dwelling, you might wish to visit a grocery store (with your measuring device in hand) to perform this part of the lab, although doing the latter might make people look at you in a curious way.

Fig. 3: Ruler with pills (*DOJ*)

Measuring and Using Linear Dimensions
(1.) Measure the height and the diameter of the cans that you have using the metric side of your ruler (use millimeters). Be sure to account for the excess metal that might exist because of a lip on the cans top or bottom when making your measurements. Record these values on the activity sheet.

(2.) Note the mass of the contents of the can, which should be written on the label. Record these masses on the activity sheet in grams.
(3.) Calculate the volume of the can using the formula V = height x pi x $(\text{diameter}/2)^2$. Record these values on the activity sheet.
(4.) Calculate the average density of the contents by dividing the mass by the volume. Record these densities on the activity sheet. Answer the questions on the sheet.

In answering the questions, keep in mind our discussion of random and systematic errors. If you want to reduce the random errors, what would you do while taking the measurements? You might wish to employ this technique in order to make sure that your answer has as little random error as possible. Even if you do minimize the random error, you will still have systematic errors. What are these sources for this experiment? How accurate do you think your density calculations of the cans' contents are, given these two sources of error? If the contents of the can you are measuring are mostly water, would you not expect the densities to be close to that of water? If the contents are much thicker than water, then what would you expect? Do you think there is any air in the can? Answer the questions on the activity sheet, and describe whether each systematic error biases the density to be either too high or too low from its actual value.

Fig. 4: Appliance Label

In part two of the lab, you will measure the amount of time that you use various electric devices around your domicile, find the power of the devices, and calculate how much energy was consumed during an average day. This will require that you keep track of all devices that you turn on for one day. The activity sheet has a table that will allow you to record this information. The power rating of the devices might require you to inspect the backside or underside of the appliances to find the consumer label that has this. For example, Figure 4 shows a label from an appliance that uses 10 W of power. A device such as a light bulb will have this information printed on the top.

Once you have measured the amount of time the device was on (in hours) and the power of the device (in W), you can calculate the amount of energy used by merely multiplying the two numbers together. This will give you the energy consumed in units of Whr. To convert this to the standard unit of kilowatt-hours (kWhr), you will need to divide the number by 1000. Thus, if you run a 100 W radio for 3 hours, then you have used 300 Whr of electricity, or 0.3 kWhr. After calculating this for each device, sum up all of the individual energy uses for the total energy used by you that day. When finished, answer the remaining questions on the sheet.

PHSC 1014 | Activity Sheet
Name: | Measurement

Table 1: Measurements of the cans

Can Contents	Can Height (cm)	Can Diameter (cm)	Content Mass (gm)	Volume (ml or cc)	Density (g/cc)

1. What are some of the random errors that occurred in your measurements (Please note that “error” here does not mean screw-up; it refers to issues with the experiment that are not accounted for by the theory)?

2. What are some of the systematic errors that occurred in your measurements?

3. Given that water has a density of 1.0 g/cc, are these values what you expect?

Table 2: Measuring Electrical Usage

Electrical Device	**Power (W)**	**Time (hrs)**	**Energy (kWhr)**
		Total	

4. If electricity costs $.06 per kWhr, then how much money did it cost you to run these appliances for one day? What if the cost had been $.10 per kWhr?