Chapter 9

Hypothesis Testing: Single Mean and Single Proportion

9.1 Hypothesis Testing: Single Mean and Single Proportion[1]

9.1.1 Student Learning Objectives

By the end of this chapter, the student should be able to:

- Differentiate between Type I and Type II Errors
- Describe hypothesis testing in general and in practice
- Conduct and interpret hypothesis tests for a single population mean, population standard deviation known.
- Conduct and interpret hypothesis tests for a single population mean, population standard deviation unknown.
- Conduct and interpret hypothesis tests for a single population proportion.

9.1.2 Introduction

One job of a statistician is to make statistical inferences about populations based on samples taken from the population. **Confidence intervals** are one way to estimate a population parameter. Another way to make a statistical inference is to make a decision about a parameter. For instance, a car dealer advertises that its new small truck gets 35 miles per gallon, on the average. A tutoring service claims that its method of tutoring helps 90% of its students get an A or a B. A company says that women managers in their company earn an average of $60,000 per year.

A statistician will make a decision about these claims. This process is called **"hypothesis testing."** A hypothesis test involves collecting data from a sample and evaluating the data. Then, the statistician makes a decision as to whether or not the data supports the claim that is made about the population.

In this chapter, you will conduct hypothesis tests on single means and single proportions. You will also learn about the errors associated with these tests.

Hypothesis testing consists of two contradictory hypotheses or statements, a decision based on the data, and a conclusion. To perform a hypothesis test, a statistician will:

[1] This content is available online at <http://cnx.org/content/m16997/1.8/>.

1. Set up two contradictory hypotheses.
2. Collect sample data (in homework problems, the data or summary statistics will be given to you).
3. Determine the correct distribution to perform the hypothesis test.
4. Analyze sample data by performing the calculations that ultimately will support one of the hypotheses.
5. Make a decision and write a meaningful conclusion.

NOTE: To do the hypothesis test homework problems for this chapter and later chapters, make copies of the appropriate special solution sheets. See the Table of Contents topic "Solution Sheets".

9.2 Null and Alternate Hypotheses[2]

The actual test begins by considering two **hypotheses**. They are called the **null hypothesis** and the **alternate hypothesis**. These hypotheses contain opposing viewpoints.

H_o: **The null hypothesis:** It is a statement about the population that will be assumed to be true unless it can be shown to be incorrect beyond a reasonable doubt.

H_a: **The alternate hypothesis:** It is a claim about the population that is contradictory to H_o and what we conclude when we reject H_o.

Example 9.1

H_o: No more than 30% of the registered voters in Santa Clara County voted in the primary election.

H_a: More than 30% of the registered voters in Santa Clara County voted in the primary election.

Example 9.2

We want to test whether the average grade point average in American colleges is 2.0 (out of 4.0) or not.

H_a: $\mu = 2.0$ H_o: $\mu \neq 2.0$

Example 9.3

We want to test if college students take less than five years to graduate from college, on the average.

H_o: $\mu \geq 5$ H_a: $\mu < 5$

Example 9.4

In an issue of **U. S. News and World Report**, an article on school standards stated that about half of all students in France, Germany, and Israel take advanced placement exams and a third pass. The same article stated that 6.6% of U. S. students take advanced placement exams and 4.4 % pass. Test if the percentage of U. S. students who take advanced placement exams is more than 6.6%.

H_o: $p = 0.066$ H_a: $p > 0.066$

Since the null and alternate hypotheses are contradictory, you must examine evidence to decide which hypothesis the evidence supports. The evidence is in the form of sample data. The sample might support either the null hypothesis or the alternate hypothesis but not both.

After you have determined which hypothesis the sample supports, you make a **decision.** There are two options for a decision. They are "reject H_o" if the sample information favors the alternate hypothesis or

[2]This content is available online at <http://cnx.org/content/m16998/1.7/>.

"do not reject H_0" if the sample information favors the null hypothesis, meaning that there is not enough information to reject the null.

Mathematical Symbols Used in H_0 and H_a:

H_0	H_a
equal ($=$)	not equal ($\neq$) **or** greater than ($>$) **or** less than ($<$)
greater than or equal to ($\geq$)	less than ($<$)
less than or equal to ($\leq$)	more than ($>$)

Table 9.1

NOTE: H_0 always has a symbol with an equal in it. H_a never has a symbol with an equal in it. The choice of symbol depends on the wording of the hypothesis test.

9.2.1 Optional Collaborative Classroom Activity

Bring to class a newspaper, some news magazines, and some Internet articles . In groups, find articles from which your group can write a null and alternate hypotheses. Discuss your hypotheses with the rest of the class.

9.3 Outcomes and the Type I and Type II Errors[3]

When you perform a hypothesis test, there are four outcomes depending on the actual truth (or falseness) of the null hypothesis H_0 and the decision to reject or not. The outcomes are summarized in the following table:

ACTION	H_0 **IS ACTUALLY**	**...**
	True	False
Do not reject H_0	Correct Outcome	Type II error
Reject H_0	Type I Error	Correct Outcome

Table 9.2

The four outcomes in the table are:

- The decision is to **not reject** H_0 when, in fact, H_0 **is true (correct decision).**
- The decision is to **reject** H_0 when, in fact, H_0 **is true** (incorrect decision known as a **Type I error**).
- The decision is to **not reject** H_0 when, in fact, H_0 **is false** (incorrect decision known as a **Type II error**).
- The decision is to **reject** H_0 when, in fact, H_0 **is false** (**correct decision** whose probability is called the **Power of the Test**).

Each of the errors occurs with a particular probability. The Greek letters α and β represent the probabilities.

α = probability of a Type I error = **P(Type I error)** = probability of rejecting the null hypothesis when the null hypothesis is true.

[3]This content is available online at <http://cnx.org/content/m17006/1.6/>.

β = probability of a Type II error = **P(Type II error)** = probability of not rejecting the null hypothesis when the null hypothesis is false.

α and β should be as small as possible because they are probabilities of errors. They are rarely 0.

The Power of the Test is $1 - \beta$. Ideally, we want a high power that is as close to 1 as possible.

The following are examples of Type I and Type II errors.

Example 9.5
Suppose the null hypothesis, H_o, is: Frank's rock climbing equipment is safe.

Type I error: Frank concludes that his rock climbing equipment may not be safe when, in fact, it really is safe. **Type II error**: Frank concludes that his rock climbing equipment is safe when, in fact, it is not safe.

α **= probability** that Frank thinks his rock climbing equipment may not be safe when, in fact, it really is. β **= probability** that Frank thinks his rock climbing equipment is safe when, in fact, it is not.

Notice that, in this case, the error with the greater consequence is the Type II error. (If Frank thinks his rock climbing equipment is safe, he will go ahead and use it.)

Example 9.6
Suppose the null hypothesis, H_o, is: The victim of an automobile accident is alive when he arrives at the emergency room of a hospital.

Type I error: The emergency crew concludes that the victim is dead when, in fact, the victim is alive. **Type II error**: The emergency crew concludes that the victim is alive when, in fact, the victim is dead.

α **= probability** that the emergency crew thinks the victim is dead when, in fact, he is really alive = *P(Type I error)*. β **= probability** that the emergency crew thinks the victim is alive when, in fact, he is dead = *P(Type II error)*.

The error with the greater consequence is the Type I error. (If the emergency crew thinks the victim is dead, they will not treat him.)

9.4 Distribution Needed for Hypothesis Testing[4]

Earlier in the course, we discussed sampling distributions. **Particular distributions are associated with hypothesis testing.** Perform tests of a population mean using a **normal distribution** or a **student-t distribution.** (Remember, use a student-t distribution when the population **standard deviation** is unknown and the population from which the sample is taken is normal.) In this chapter we perform tests of a population proportion using a normal distribution (usually n is large or the sample size is large).

If you are testing a **single population mean**, the distribution for the test is for **averages**:

$$\overline{X} \sim N\left(\mu_X, \frac{\sigma_X}{\sqrt{n}}\right) \qquad \text{or} \qquad t_{df}$$

The population parameter is μ. The estimated value (point estimate) for μ is $\overline{x}$, the sample mean.

[4]This content is available online at <http://cnx.org/content/m17017/1.6/>.

If you are testing a **single population proportion**, the distribution for the test is for proportions or percentages:

$$P' \sim N\left(p, \sqrt{\frac{p \cdot q}{n}}\right)$$

The population parameter is p. The estimated value (point estimate) for p is p'. $p' = \frac{x}{n}$ where x is the number of successes and n is the sample size.

9.5 Assumption[5]

When you perform a **hypothesis test of a single population mean** μ using a **Student-t distribution** (often called a t-test), there are fundamental assumptions that need to be met in order for the test to work properly. Your data should be a **simple random sample** that comes from a population that is approximately **normally distributed**. You use the sample **standard deviation** to approximate the population standard deviation. (Note that if the sample size is larger than 30, a t-test will work even if the population is not approximately normally distributed).

When you perform a **hypothesis test of a single population mean** μ using a normal distribution (often called a z-test), you take a simple random sample from the population. The population you are testing is normally distributed or your sample size is larger than 30 or both. You know the value of the population standard deviation.

When you perform a **hypothesis test of a single population proportion** p, you take a simple random sample from the population. You must meet the conditions for a **binomial distribution** which are there are a certain number n of independent trials, the outcomes of any trial are success or failure, and each trial has the same probability of a success p. The shape of the binomial distribution needs to be similar to the shape of the normal distribution. To ensure this, the quantities np and nq must both be greater than five ($np > 5$ and $nq > 5$). Then the binomial distribution of sample (estimated) proportion can be approximated by the normal distribution with $\mu = p$ and $\sigma = \sqrt{\frac{p \cdot q}{n}}$. Remember that $q = 1 - p$.

9.6 Rare Events[6]

Suppose you make an assumption about a property of the population (this assumption is the **null hypothesis**). Then you gather sample data randomly. If the sample has properties that would be very **unlikely** to occur if the assumption is true, then you would conclude that your assumption about the population is probably incorrect. (Remember that your assumption is just an **assumption** - it is not a fact and it may or may not be true. But your sample data is real and it is showing you a fact that seems to contradict your assumption.)

For example, Didi and Ali are at a birthday party of a very wealthy friend. They hurry to be first in line to grab a prize from a tall basket that they cannot see inside because they will be blindfolded. There are 200 plastic bubbles in the basket and Didi and Ali have been told that there is only one with a $100 bill. Didi is the first person to reach into the basket and pull out a bubble. Her bubble contains a $100 bill. The probability of this happening is $\frac{1}{200} = 0.005$. Because this is so unlikely, Ali is hoping that what the two of them were told is wrong and there are more $100 bills in the basket. A "rare event" has occurred (Didi getting the $100 bill) so Ali doubts the assumption about only one $100 bill being in the basket.

[5] This content is available online at <http://cnx.org/content/m17002/1.7/>.
[6] This content is available online at <http://cnx.org/content/m16994/1.5/>.

9.7 Using the Sample to Support One of the Hypotheses[7]

Use the sample (data) to calculate the actual probability of getting the test result, called the **p-value**. The p-value is the **probability that an outcome of the data (for example, the sample mean) will happen purely by chance when the null hypothesis is true**.

A large p-value calculated from the data indicates that the sample result is likely happening purely by chance. The data supports the **null hypothesis** so we do not reject it. The smaller the p-value, the more unlikely the outcome, and the stronger the evidence is against the null hypothesis. We would reject the null hypothesis if the evidence is strongly against the null hypothesis.

The p-value is sometimes called the **computed** α because it is calculated from the data. You can think of it as the probability of (incorrectly) rejecting the null hypothesis when the null hypothesis is actually true.

Draw a graph that shows the p-value. The hypothesis test is easier to perform if you use a graph because you see the problem more clearly.

Example 9.7: (to illustrate the p-value)
Suppose a baker claims that his bread height is more than 15 cm, on the average. Several of his customers do not believe him. To persuade his customers that he is right, the baker decides to do a hypothesis test. He bakes 10 loaves of bread. The average height of the sample loaves is 17 cm. The baker knows from baking hundreds of loaves of bread that the **standard deviation** for the height is 0.5 cm.

The null hypothesis could be H_o: $\mu \leq 15$ The alternate hypothesis is H_a: $\mu > 15$

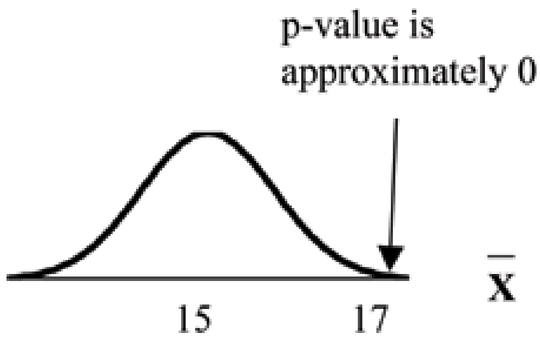
The words **"is more than"** translates as a "$>$ " so "$\mu > 15$" goes into the alternate hypothesis. The null hypothesis must contradict the alternate hypothesis.

Since σ **is known** ($\sigma = 0.5$ cm.), the distribution for the test is normal with mean $\mu = 15$ and standard deviation $\frac{\sigma}{\sqrt{n}} = \frac{0.5}{\sqrt{10}} = 0.16$.

Suppose the null hypothesis is true (the average height of the loaves is no more than 15 cm). Then is the average height (17 cm) calculated from the sample unexpectedly large? The hypothesis test works by asking the question how **unlikely** the sample average would be if the null hypothesis were true. The graph shows how far out the sample average is on the normal curve. How far out the sample average is on the normal curve is measured by the p-value. The p-value is the probability that, if we were to take other samples, any other sample average would fall at least as far out as 17 cm.

The p-value, then, is the probability that a sample average is the same or greater than 17 cm. when the population mean is, in fact, 15 cm. We can calculate this probability using the normal distribution for averages from Chapter 7.

[7] This content is available online at <http://cnx.org/content/m16995/1.8/>.



p-value $= P\left(\overline{X} > 17\right)$ which is approximately 0.

A p-value of approximately 0 tells us that it is highly unlikely that a loaf of bread rises no more than 15 cm, on the average. That is, almost 0% of all loaves of bread would be at least as high as 17 cm. **purely by CHANCE**. Because the outcome of 17 cm. is so **unlikely (meaning it is happening NOT by chance alone)**, we conclude that the evidence is strongly against the null hypothesis (the average height is at most 15 cm.). There is sufficient evidence that the true average height for the population of the baker's loaves of bread is greater than 15 cm.

9.8 Decision and Conclusion[8]

A systematic way to make a decision of whether to reject or not reject the **null hypothesis** is to compare the **p-value** and a **preset or preconceived** α **(also called a "significance level")**. A preset α is the probability of a **Type I error** (rejecting the null hypothesis when the null hypothesis is true). It may or may not be given to you at the beginning of the problem.

When you make a **decision** to reject or not reject H_o, do as follows:

- If $\alpha >$ *p-value*, reject H_o. The results of the sample data are significant. There is sufficient evidence to conclude that H_o is an incorrect belief and that the **alternative hypothesis**, H_a, may be correct.
- If $\alpha \leq$ *p-value*, do not reject H_o. The results of the sample data are not significant. There is not sufficient evidence to conclude that the alternative hypothesis, H_a, may be correct.
- When you "do not reject H_o", it does not mean that you should believe that H_o is true. It simply means that the sample data has **failed** to provide sufficient evidence to cast serious doubt about the truthfulness of H_o.

Conclusion: After you make your decision, write a thoughtful **conclusion** about the hypotheses in terms of the given problem.

9.9 Additional Information[9]

- In a **hypothesis test** problem, you may see words such as "the level of significance is 1%." The "1%" is the preconceived or preset α.
- The statistician setting up the hypothesis test selects the value of α to use **before** collecting the sample data.
- **If no level of significance is given, we generally can use** $\alpha = 0.05$**.**

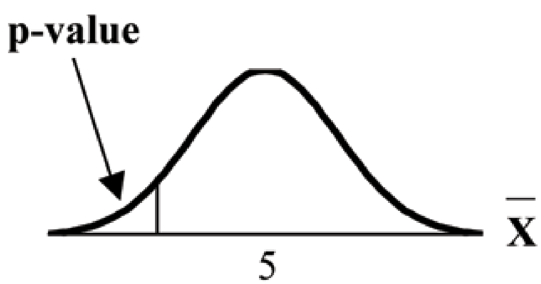
[8]This content is available online at <http://cnx.org/content/m16992/1.7/>.
[9]This content is available online at <http://cnx.org/content/m16999/1.6/>.

- When you calculate the **p-value** and draw the picture, the p-value is in the left tail, the right tail, or split evenly between the two tails. For this reason, we call the hypothesis test left, right, or two tailed.
- The **alternate hypothesis**, H_a, tells you if the test is left, right, or two-tailed. It is the **key** to conducting the appropriate test.
- H_a **never** has a symbol that contains an equal sign.

The following examples illustrate a left, right, and two-tailed test.

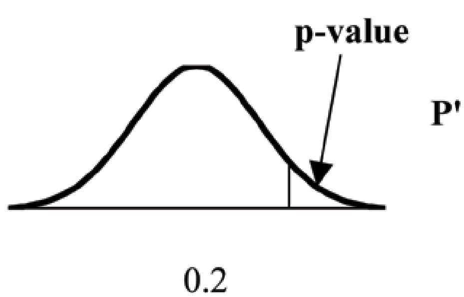
Example 9.8
H_o: $\mu = 5$ H_a: $\mu < 5$

Test of a single population mean. H_a tells you the test is left-tailed. The picture of the p-value is as follows:



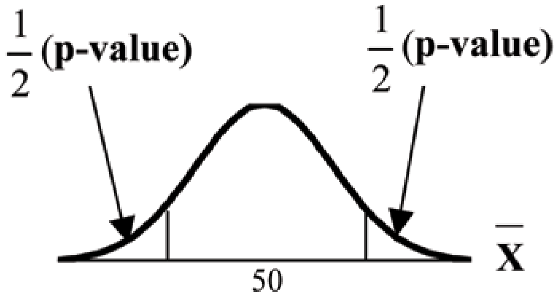
Example 9.9
H_o: $p \leq 0.2$ H_a: $p > 0.2$

This is a test of a single population proportion. H_a tells you the test is **right-tailed**. The picture of the p-value is as follows:



Example 9.10
H_o: $\mu = 50$ H_a: $\mu \neq 50$

This is a test of a single population mean. H_a tells you the test is **two-tailed**. The picture of the p-value is as follows.



$\frac{1}{2}$ (p-value)
$\frac{1}{2}$ (p-value)
50
$\overline{X}$

9.10 Summary of the Hypothesis Test[10]

The **hypothesis test** itself has an established process. This can be summarized as follows:

1. Determine H_o and H_a. Remember, they are contradictory.
2. Determine the random variable.
3. Determine the distribution for the test.
4. Draw a graph, calculate the test statistic, and use the test statistic to calculate the **p-value**. (A z-score and a t-score are examples of test statistics.)
5. Compare the preconceived α with the p-value, make a decision (reject or cannot reject H_o), and write a clear conclusion using English sentences.

Notice that in performing the hypothesis test, you use α and not β. β is needed to help determine the sample size of the data that is used in calculating the p-value. Remember that the quantity $1-\beta$ is called the **Power of the Test**. A high power is desirable. If the power is too low, statisticians typically increase the sample size while keeping α the same. If the power is low, the null hypothesis might not be rejected when it should be.

9.11 Examples[11]

Example 9.11
Jeffrey, as an eight-year old, **established an average time of 16.43 seconds** for swimming the 25-yard freestyle, with a **standard deviation of 0.8 seconds**. His dad, Frank, thought that Jeffrey could swim the 25-yard freestyle faster by using goggles. Frank bought Jeffrey a new pair of expensive goggles and timed Jeffrey for **15 25-yard freestyle swims**. For the 15 swims, **Jeffrey's average time was 16 seconds. Frank thought that the goggles helped Jeffrey to swim faster than the 16.43 seconds.** Conduct a hypothesis test using a preset $\alpha = 0.05$. Assume that the swim times for the 25-yard freestyle are normal.

Solution
Set up the Hypothesis Test:

Since the problem is about a mean (average), this is a **test of a single population mean**.

H_o: $\mu = 16.43$ $\quad$ H_a: $\mu < 16.43$

For Jeffrey to swim faster, his time will be less than 16.43 seconds. The "<" tells you this is left-tailed.

Determine the distribution needed:

Random variable: $\overline{X}$ = the average time to swim the 25-yard freestyle.

Distribution for the test: $\overline{X}$ is normal (population **standard deviation** is known: $\sigma = 0.8$)

$\overline{X} \sim N\left(\mu, \frac{\sigma_X}{\sqrt{n}}\right)$ $\quad$ Therefore, $\overline{X} \sim N\left(16.43, \frac{0.8}{\sqrt{15}}\right)$

$\mu = 16.43$ comes from H_0 and not the data. $\sigma = 0.8$, and $n = 15$.

Calculate the p-value using the normal distribution for a mean:

[10]This content is available online at <http://cnx.org/content/m16993/1.3/>.
[11]This content is available online at <http://cnx.org/content/m17005/1.12/>.

p-value $= P\left(\bar{X} < 16\right) = 0.0187$ where the sample mean in the problem is given s 16.

p-value $= 0.0187$ (This is called the **actual level of significance**.) The p-value is the area to the left of the sample mean is given as 16.

Graph:

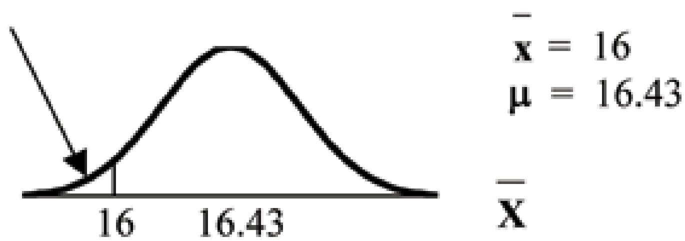


Figure 9.1

$\mu = 16.43$ comes from H_o. Our assumption is $\mu = 16.43$.

Interpretation of the p-value: If H_o **is true**, there is a 0.0187 probability (1.87%) that Jeffrey's mean (or average) time to swim the 25-yard freestyle is 16 seconds or less. Because a 1.87% chance is small, the mean time of 16 seconds or less is not happening randomly. It is a rare event.

Compare α and the p-value:

$\alpha = 0.05$ $\quad$ *p-value* $= 0.0187$ $\quad$ $\alpha >$ *p-value*

Make a decision: Since $\alpha >$ *p-value*, reject H_o.

This means that you reject $\mu = 16.43$. In other words, you do not think Jeffrey swims the 25-yard freestyle in 16.43 seconds but faster with the new goggles.

Conclusion: At the 5% significance level, we conclude that Jeffrey swims faster using the new goggles. The sample data show there is sufficient evidence that Jeffrey's mean time to swim the 25-yard freestyle is less than 16.43 seconds.

The p-value can easily be calculated using the TI-83+ and the TI-84 calculators:

Press `STAT` and arrow over to `TESTS`. Press `1:Z-Test`. Arrow over to `Stats` and press `ENTER`. Arrow down and enter 16.43 for μ_0 (null hypothesis), .8 for σ, 16 for the sample mean, and 15 for n. Arrow down to μ: (alternate hypothesis) and arrow over to $<\mu_0$. Press `ENTER`. Arrow down to `Calculate` and press `ENTER`. The calculator not only calculates the p-value ($p = 0.0187$) but it also calculates the test statistic (z-score) for the sample mean. $\mu < 16.43$ is the alternate hypothesis. Do this set of instructions again except arrow to `Draw` (instead of `Calculate`). Press `ENTER`. A shaded graph appears with $z = -2.08$ (test statistic) and $p = 0.0187$ (p-value). Make sure when you use `Draw` that no other equations are highlighted in $Y =$ and the plots are turned off.

When the calculator does a Z-Test, the `Z-Test` function finds the p-value by doing a normal probability calculation using the **Central Limit Theorem**:

$P\left(\overline{X} < 16 = \texttt{2nd DISTR normcdf}\left(-10^\wedge 99, 16, 16.43, 0.8/\sqrt{15}\right)\right.$.

The Type I and Type II errors for this problem are as follows:

The Type I error is to conclude that Jeffrey swims the 25-yard freestyle, on average, in less than 16.43 seconds when, in fact, he actually swims the 25-yard freestyle, on average, in 16.43 seconds. (Reject the null hypothesis when the null hypothesis is true.)

The Type II error is to conclude that Jeffrey swims the 25-yard freestyle, on average, in 16.43 seconds when, in fact, he actually swims the 25-yard freestyle, on average, in less than 16.43 seconds. (Do not reject the null hypothesis when the null hypothesis is false.)

Historical Note: The traditional way to compare the two probabilities, α and the p-value, is to compare their test statistics (z-scores). The calculated test statistic for the p-value is -2.08. (From the Central Limit Theorem, the test statistic formula is $z = \frac{\overline{x}-\mu_X}{\left(\frac{\sigma_X}{\sqrt{n}}\right)}$. For this problem, $\overline{x} = 16$, $\mu_X = 16.43$ from the null hypothesis, $\sigma_X = 0.8$, and $n = 15$.) You can find the test statistic for $\alpha = 0.05$ in the normal table (see **15.Tables** in the Table of Contents). The z-score for an area to the left equal to 0.05 is midway between -1.65 and -1.64 (0.05 is midway between 0.0505 and 0.0495). The z-score is -1.645. Since $-1.645 > -2.08$ (which demonstrates that $\alpha >$ *p-value*), reject H_o. Traditionally, the decision to reject or not reject was done in this way. Today, comparing the two probabilities α and the p-value is very common and advantageous. For this problem, the p-value, 0.0187 is considerably smaller than α, 0.05. You can be confident about your decision to reject. It is difficult to know that the p-value is traditionally smaller than α by just examining the test statistics. The graph shows α, the p-value, and the two test statistics (z scores).

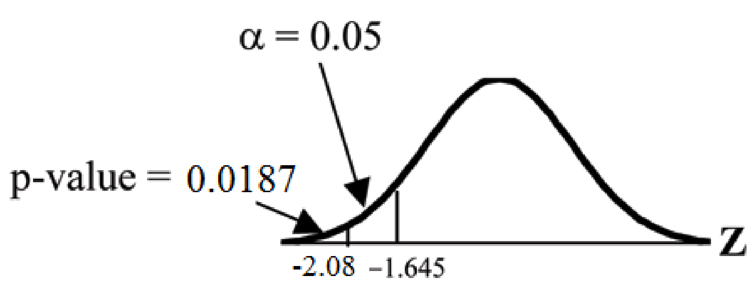


Figure 9.2

Example 9.12

A college football coach thought that his players could bench press an **average of 275 pounds**. It is known that the **standard deviation is 55 pounds**. Three of his players thought that the average was **more than** that amount. They asked **30** of their teammates for their estimated maximum lift on the bench press exercise. The data ranged from 205 pounds to 385 pounds. The actual different weights were (frequencies are in parentheses) 205(3); 215(3); 225(1); 241(2); 252(2); 265(2); 275(2); 313(2); 316(5); 338(2); 341(1); 345(2); 368(2); 385(1). (Source: data from Reuben Davis, Kraig Evans, and Scott Gunderson.)

Conduct a hypothesis test using a 2.5% level of significance to determine if the bench press average is **more than 275 pounds**.

Solution
Set up the Hypothesis Test:

Since the problem is about a mean (average), this is a **test of a single population mean**.

H_o: $\mu = 275$ H_a: $\mu > 275$ This is a right-tailed test.

Calculating the distribution needed:

Random variable: $\overline{X}$ = the average weight lifted by the football players.

Distribution for the test: It is normal because σ is known.

$\overline{X} \sim N\left(275, \frac{55}{\sqrt{30}}\right)$

$\overline{x} = 286.2$ pounds (from the data).

$\sigma = 55$ pounds **(Always use σ if you know it.)** We assume $\mu = 275$ pounds unless our data shows us otherwise.

Calculate the p-value using the normal distribution for a mean:

$p\text{-}value = P(\ \overline{X} > 286.2 = 0.1323$ where the sample mean is calculated as 286.2 pounds from the data.

Interpretation of the p-value: If H_o is true, then there is a 0.1323 probability (13.23%) that the football players can lift a mean (or average) weight of 286.2 pounds or more. Because a 13.23% chance is large enough, a mean weight lift of 286.2 pounds or more is happening randomly and is not a rare event.

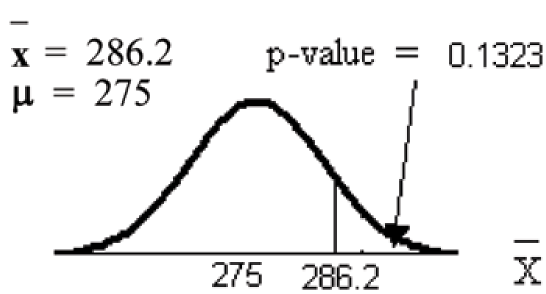


Figure 9.3

Compare α and the p-value:

$\alpha = 0.025$ $p\text{-}value = 0.1323$

Make a decision: Since $\alpha < p\text{-}value$, do not reject H_o.

Conclusion: At the 2.5% level of significance, from the sample data, there is not sufficient evidence to conclude that the true mean weight lifted is more than 275 pounds.

The p-value can easily be calculated using the TI-83+ and the TI-84 calculators:

Put the data and frequencies into lists. Press `STAT` and arrow over to `TESTS`. Press `1:Z-Test`. Arrow over to `Data` and press `ENTER`. Arrow down and enter 275 for μ_0, 55 for σ, the name of the list where you put the data, and the name of the list where you put the frequencies. Arrow down to μ : and arrow over to $> \mu_0$. Press `ENTER`. Arrow down to `Calculate` and press `ENTER`. The calculator not only calculates the p-value ($p = 0.1331$, a little different from the above calculation - in it we used the sample mean rounded to one decimal place instead of the data) but it also calculates the test statistic (z-score) for the sample mean, the sample mean, and the sample standard deviation. $\mu > 275$ is the alternate hypothesis. Do this set of instructions again except arrow to `Draw` (instead of `Calculate`). Press `ENTER`. A shaded graph appears with $z = 1.112$ (test statistic) and $p = 0.1331$ (p-value). Make sure when you use `Draw` that no other equations are highlighted in $Y =$ and the plots are turned off.

Example 9.13
Statistics students believe that the average score on the first statistics test is 65. A statistics instructor thinks the average score is higher than 65. He samples ten statistics students and obtains the scores 65; 65; 70; 67; 66; 63; 63; 68; 72; 71. He performs a hypothesis test using a 5% level of significance. The data are from a normal distribution.

Solution
Set up the Hypothesis Test:

A 5% level of significance means that $\alpha = 0.05$. This is a test of a **single population mean**.

H_o: $\mu = 65$ $\qquad$ H_a: $\mu > 65$

Since the instructor thinks the average score is higher, use a "$>$ ". The "$>$ " means the test is right-tailed.

Determine the distribution needed:

Random variable: $\overline{X}$ = average score on the first statistics test.

Distribution for the test: If you read the problem carefully, you will notice that there is **no population standard deviation given**. You are only given $n = 10$ sample data values. Notice also that the data come from a normal distribution. This means that the distribution for the test is a student-t.

Use t_{df}. Therefore, the distribution for the test is t_9 where $n = 10$ and $df = 10 - 1 = 9$.

Calculate the p-value using the Student-t distribution:

$p\text{-}value = P\,(\ \overline{X} > 67 = 0.0396$ where the sample mean and sample standard deviation are calculated as 67 and 3.1972 from the data.

Interpretation of the p-value: If the null hypothesis is true, then there is a 0.0396 probability (3.96%) that the sample mean is 67 pounds or more.

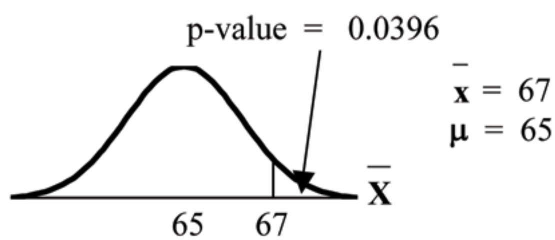


Figure 9.4

Compare α and the p-value:

Since $\alpha = .05$ and $p\text{-}value = 0.0396$. Therefore, $\alpha > p\text{-}value$.

Make a decision: Since $\alpha > p\text{-}value$, reject H_o.

This means you reject $\mu = 65$. In other words, you believe the average test score is more than 65.

Conclusion: At a 5% level of significance, the sample data show sufficient evidence that the mean (average) test score is more than 65, just as the math instructor thinks.

The p-value can easily be calculated using the TI-83+ and the TI-84 calculators:

Put the data into a list. Press `STAT` and arrow over to `TESTS`. Press `2:T-Test`. Arrow over to `Data` and press `ENTER`. Arrow down and enter 65 for μ_0, the name of the list where you put the data, and 1 for `Freq:`. Arrow down to μ : and arrow over to $> \mu_0$. Press `ENTER`. Arrow down to `Calculate` and press `ENTER`. The calculator not only calculates the p-value ($p = 0.0396$) but it also calculates the test statistic (t-score) for the sample mean, the sample mean, and the sample standard deviation. $\mu > 65$ is the alternate hypothesis. Do this set of instructions again except arrow to `Draw` (instead of `Calculate`). Press `ENTER`. A shaded graph appears with $t = 1.9781$ (test statistic) and $p = 0.0396$ (p-value). Make sure when you use `Draw` that no other equations are highlighted in $Y =$ and the plots are turned off.

Example 9.14
Joon believes that 50% of first-time brides in the United States are younger than their grooms. She performs a hypothesis test to determine if the percentage is **the same or different from 50%**. Joon samples **100 first-time brides** and **53** reply that they are younger than their grooms. For the hypothesis test, she uses a 1% level of significance.

Solution
Set up the Hypothesis Test:

The 1% level of significance means that $\alpha = 0.01$. This is a **test of a single population proportion**.

H_o: $p = 0.50$ $\qquad$ H_a: $p \neq 0.50$

The words **"is the same or different from"** tell you this is a two-tailed test.

Calculate the distribution needed:

Random variable: P' = the percent of of first-time brides who are younger than their grooms. Distribution

Distribution for the test: The problem contains no mention of an average. The information is given in terms of percentages. Use the distribution for P', the estimated proportion.

$P' \sim N\left(p, \sqrt{\frac{p \cdot q}{n}}\right)$ Therefore, $P' \sim N\left(0.5, \sqrt{\frac{0.5 \cdot 0.5}{100}}\right)$ where $p = 0.50$, $q = 1 - p = 0.50$, and $n = 100$.

Calculate the p-value using the normal distribution for proportions:

$p\text{-}value = P\,(P' < 0.47 \text{ or } P' > 0.53 = 0.5485$

where $x = 53$, $p' = \frac{x}{n} = \frac{53}{100} = 0.53$.

Interpretation of the p-value: If the null hypothesis is true, there is 0.5485 probability (54.85%) that the sample (estimated) proportion p' is 0.53 or more OR 0.47 or less (see the graph below).

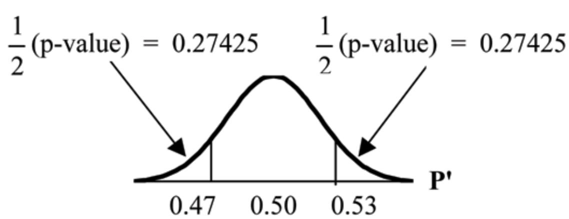


Figure 9.5

$\mu = p = 0.50$ comes from H_o, the null hypothesis.

p'= 0.53. Since the curve is symmetrical and the test is two-tailed, the p' for the left tail is equal to $0.50 - 0.03 = 0.47$ where $\mu = p = 0.50$. (0.03 is the difference between 0.53 and 0.50.)

Compare α and the p-value:

Since $\alpha = 0.01$ and $p\text{-}value = 0.5485$. Therefore, $\alpha < p\text{-}value$.

Make a decision: Since $\alpha < p\text{-}value$, you cannot reject H_o.

Conclusion: At the 1% level of significance, the sample data do not show sufficient evidence that the percentage of first-time brides who are younger than their grooms is different from 50%.

The p-value can easily be calculated using the TI-83+ and the TI-84 calculators:

Press `STAT` and arrow over to `TESTS`. Press `5:1-PropZTest`. Enter .5 for p_0 and 100 for n. Arrow down to `Prop` and arrow to `not equals` p_0. Press `ENTER`. Arrow down to `Calculate` and press `ENTER`. The calculator calculates the p-value ($p = 0.5485$) and the test statistic (z-score). `Prop not equals` .5 is the alternate hypothesis. Do this set of instructions again except arrow to `Draw` (instead of `Calculate`). Press `ENTER`. A shaded graph appears with $z = 0.6$ (test statistic) and $p = 0.5485$

(p-value). Make sure when you use `Draw` that no other equations are highlighted in $Y =$ and the plots are turned off.

The Type I and Type II errors are as follows:

The Type I error is to conclude that the proportion of first-time brides that are younger than their grooms is different from 50% when, in fact, the proportion is actually 50%. (Reject the null hypothesis when the null hypothesis is true).

The Type II error is to conclude that the proportion of first-time brides that are younger than their grooms is equal to 50% when, in fact, the proportion is different from 50%. (Do not reject the null hypothesis when the null hypothesis is false.)

Example 9.15
Problem 1
Suppose a consumer group suspects that the proportion of households that have three cell phones is not known to be 30%. A cell phone company has reason to believe that the proportion is 30%. Before they start a big advertising campaign, they conduct a hypothesis test. Their marketing people survey 150 households with the result that 43 of the households have three cell phones.

Solution
Set up the Hypothesis Test:

H_o: $p = 0.30$ H_a: $p \neq 0.30$

Determine the distribution needed:

The **random variable** is P' = proportion of households that have three cell phones.

The **distribution** for the hypothesis test is $P' \sim N\left(0.30, \sqrt{\frac{0.30 \cdot 0.70}{150}}\right)$

Problem 2 *(Solution on p. 380.)*
The value that helps determine the p-value is p'. Calculate p'.

Problem 3 *(Solution on p. 380.)*
What is a **success** for this problem?

Problem 4 *(Solution on p. 380.)*
What is the level of significance?

Draw the graph for this problem. Draw the horizontal axis. Label and shade appropriately.

Problem 5 *(Solution on p. 380.)*
Calculate the p-value.

Problem 6 *(Solution on p. 380.)*
Make a decision. _____________(Reject/Do not reject) H_0 because____________.

The next example is a poem written by a statistics student named Nicole Hart. The solution to the problem follows the poem. Notice that the hypothesis test is for a single population proportion. This means that the null and alternate hypotheses use the parameter p. The distribution for the test is normal. The estimated proportion p' is the proportion of fleas killed to the total fleas found on Fido. This is sample information. The problem gives a preconceived $\alpha = 0.01$, for comparison, and a 95% confidence interval computation. The poem is clever and humorous, so please enjoy it!

NOTE: Notice the solution sheet that has the solution. Look in the Table of Contents for the topic "Solution Sheets." Use copies of the appropriate solution sheet for homework problems.

Example 9.16

```
    My dog has so many fleas,
They do not come off with ease.
As for shampoo, I have tried many types
Even one called Bubble Hype,
Which only killed 25% of the fleas,
Unfortunately I was not pleased.

I've used all kinds of soap,
Until I had give up hope
Until one day I saw
An ad that put me in awe.

A shampoo used for dogs
Called GOOD ENOUGH to Clean a Hog
Guaranteed to kill more fleas.

I gave Fido a bath
And after doing the math
His number of fleas
Started dropping by 3's!

Before his shampoo
I counted 42.
At the end of his bath,
I redid the math
And the new shampoo had killed 17 fleas.
So now I was pleased.

Now it is time for you to have some fun
With the level of significance being .01,
You must help me figure out
Use the new shampoo or go without?
```

Solution
Set up the Hypothesis Test:

H_o: $p = 0.25$ H_a: $p > 0.25$

Determine the distribution needed:

In words, CLEARLY state what your random variable $\overline{X}$ or P' represents.

P' = The proportion of fleas that are killed by the new shampoo

State the distribution to use for the test.

Normal: $N\left(0.25, \sqrt{\frac{(0.25)(1-0.25)}{42}}\right)$

Test Statistic: $z = 2.3163$

Calculate the p-value using the normal distribution for proportions:

p-value $= 0.0103$

In 1 – 2 complete sentences, explain what the p-value means for this problem.

If the null hypothesis is true (the proportion is 0.25), then there is a 0.0103 probability that the sample (estimated) proportion is 0.4048 $\left(\frac{17}{42}\right)$ or more.

Use the previous information to sketch a picture of this situation. CLEARLY, label and scale the horizontal axis and shade the region(s) corresponding to the p-value.

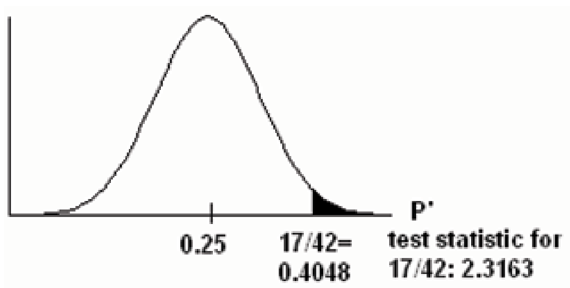


Figure 9.6

Compare α and the p-value:

Indicate the correct decision ("reject" or "do not reject" the null hypothesis), the reason for it, and write an appropriate conclusion, using COMPLETE SENTENCES.

alpha	decision	reason for decision
0.01	Do not reject H_o	$\alpha < p\text{-value}$

Table 9.3

Conclusion: At the 1% level of significance, the sample data do not show sufficient evidence that the percentage of fleas that are killed by the new shampoo is more than 25%.

Construct a 95% Confidence Interval for the true mean or proportion. Include a sketch of the graph of the situation. Label the point estimate and the lower and upper bounds of the Confidence Interval.

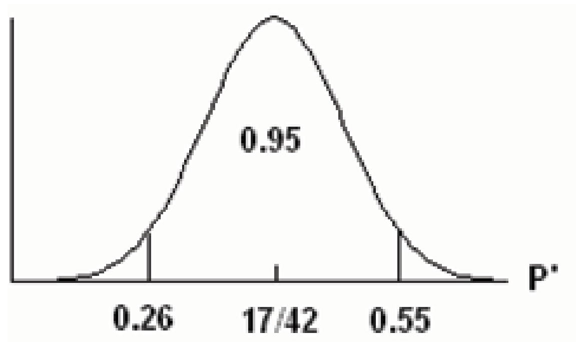


Figure 9.7

Confidence Interval: $(0.26, 0.55)$ We are 95% confident that the true population proportion p of fleas that are killed by the new shampoo is between 26% and 55%.

NOTE: This test result is not very definitive since the p-value is very close to alpha. In reality, one would probably do more tests by giving the dog another bath after the fleas have had a chance to return.

9.12 Summary of Formulas[12]

H_o and H_a are contradictory.

If H_o has:	equal $(=)$	greater than or equal to $(\geq)$	less than or equal to $(\leq)$
then H_a has:	not equal $(\neq)$ **or** greater than $(>)$ **or** less than $(<)$	less than $(<)$	greater than $(>)$

Table 9.4

If $\alpha \leq$ p-value, then do not reject H_o.

If $\alpha >$ p-value, then reject H_o .

α is preconceived. Its value is set before the hypothesis test starts. The p-value is calculated from the data.

α = probability of a Type I error = P(Type I error) = probability of rejecting the null hypothesis when the null hypothesis is true.

β = probability of a Type II error = P(Type II error) = probability of not rejecting the null hypothesis when the null hypothesis is false.

If there is no given preconceived α, then use $\alpha = 0.05$.

Types of Hypothesis Tests

- Single population mean, **known** population variance (or standard deviation): **Normal test**.
- Single population mean, **unknown** population variance (or standard deviation): **Student-t test**.
- Single population proportion: **Normal test**.

[12]This content is available online at <http://cnx.org/content/m16996/1.7/>.

9.13 Practice 1: Single Mean, Known Population Standard Deviation[13]

9.13.1 Student Learning Outcomes

- The student will explore hypothesis testing with single mean and known population standard deviation data.

9.13.2 Given

Suppose that a recent article stated that the average time spent in jail by a first–time convicted burglar is 2.5 years. A study was then done to see if the average time has increased in the new century. A random sample of 26 first–time convicted burglars in a recent year was picked. The average length of time in jail from the survey was 3 years with a standard deviation of 1.8 years. Suppose that it is somehow known that the population standard deviation is 1.5. Conduct a hypothesis test to determine if the average length of jail time has increased.

9.13.3 Hypothesis Testing: Single Average

Exercise 9.13.1 ***(Solution on p. 380.)***
Is this a test of averages or proportions?

Exercise 9.13.2 ***(Solution on p. 380.)***
State the null and alternative hypotheses.

a. H_o:
b. H_a:

Exercise 9.13.3 ***(Solution on p. 380.)***
Is this a right-tailed, left-tailed, or two-tailed test? How do you know?

Exercise 9.13.4 ***(Solution on p. 380.)***
What symbol represents the Random Variable for this test?

Exercise 9.13.5 ***(Solution on p. 380.)***
In words, define the Random Variable for this test.

Exercise 9.13.6 ***(Solution on p. 380.)***
Is the population standard deviation known and, if so, what is it?

Exercise 9.13.7 ***(Solution on p. 380.)***
Calculate the following:

a. $\overline{x} =$
b. $\sigma =$
c. $s_x =$
d. $n =$

Exercise 9.13.8 ***(Solution on p. 380.)***
Since both σ and s_x are given, which should be used? In 1 -2 complete sentences, explain why.

Exercise 9.13.9 ***(Solution on p. 380.)***
State the distribution to use for the hypothesis test.

Exercise 9.13.10
Sketch a graph of the situation. Label the horizontal axis. Mark the hypothesized mean and the sample mean $\overline{x}$. Shade the area corresponding to the p-value.

[13] This content is available online at <http://cnx.org/content/m17004/1.8/>.

Exercise 9.13.11 *(Solution on p. 380.)*

Find the p-value.

Exercise 9.13.12 *(Solution on p. 380.)*

At a pre-conceived $\alpha = 0.05$, what is your:

- **a.** Decision:
- **b.** Reason for the decision:
- **c.** Conclusion (write out in a complete sentence):

9.13.4 Discussion Questions

Exercise 9.13.13

Does it appear that the average jail time spent for first time convicted burglars has increased? Why or why not?

9.14 Practice 2: Single Mean, Unknown Population Standard Deviation[14]

9.14.1 Student Learning Outcomes

- The student will explore the properties of hypothesis testing with a single mean and unknown population standard deviation.

9.14.2 Given

A random survey of 75 death row inmates revealed that the average length of time on death row is 17.4 years with a standard deviation of 6.3 years. Conduct a hypothesis test to determine if the population average time on death row could likely be 15 years.

9.14.3 Hypothesis Testing: Single Average

Exercise 9.14.1 *(Solution on p. 381.)*
Is this a test of averages or proportions?

Exercise 9.14.2 *(Solution on p. 381.)*
State the null and alternative hypotheses.

a. H_o :
b. H_a :

Exercise 9.14.3 *(Solution on p. 381.)*
Is this a right-tailed, left-tailed, or two-tailed test? How do you know?

Exercise 9.14.4 *(Solution on p. 381.)*
What symbol represents the Random Variable for this test?

Exercise 9.14.5 *(Solution on p. 381.)*
In words, define the Random Variable for this test.

Exercise 9.14.6 *(Solution on p. 381.)*
Is the population standard deviation known and, if so, what is it?

Exercise 9.14.7 *(Solution on p. 381.)*
Calculate the following:

a. $\overline{x} =$
b. $6.3 =$
c. $n =$

Exercise 9.14.8 *(Solution on p. 381.)*
Which test should be used? In 1 -2 complete sentences, explain why.

Exercise 9.14.9 *(Solution on p. 381.)*
State the distribution to use for the hypothesis test.

Exercise 9.14.10
Sketch a graph of the situation. Label the horizontal axis. Mark the hypothesized mean and the sample mean, $\overline{x}$. Shade the area corresponding to the p-value.

[14]This content is available online at <http://cnx.org/content/m17016/1.8/>.

Figure 9.8

Exercise 9.14.11 *(Solution on p. 381.)*
Find the p-value.

Exercise 9.14.12 *(Solution on p. 381.)*
At a pre-conceived $\alpha = 0.05$, what is your:

- **a.** Decision:
- **b.** Reason for the decision:
- **c.** Conclusion (write out in a complete sentence):

9.14.4 Discussion Question

Does it appear that the average time on death row could be 15 years? Why or why not?

9.15 Practice 3: Single Proportion[15]

9.15.1 Student Learning Outcomes

- The student will explore the properties of hypothesis testing with a single proportion.

9.15.2 Given

The National Institute of Mental Health published an article stating that in any one-year period, approximately 9.5 percent of American adults suffer from depression or a depressive illness. (http://www.nimh.nih.gov/publicat/depression.cfm) Suppose that in a survey of 100 people in a certain town, seven of them suffered from depression or a depressive illness. Conduct a hypothesis test to determine if the true proportion of people in that town suffering from depression or a depressive illness is lower than the percent in the general adult American population.

9.15.3 Hypothesis Testing: Single Proportion

Exercise 9.15.1 *(Solution on p. 381.)*
Is this a test of averages or proportions?

Exercise 9.15.2 *(Solution on p. 381.)*
State the null and alternative hypotheses.

a. H_0 :
b. H_a :

Exercise 9.15.3 *(Solution on p. 381.)*
Is this a right-tailed, left-tailed, or two-tailed test? How do you know?

Exercise 9.15.4 *(Solution on p. 381.)*
What symbol represents the Random Variable for this test?

Exercise 9.15.5 *(Solution on p. 381.)*
In words, define the Random Variable for this test.

Exercise 9.15.6 *(Solution on p. 381.)*
Calculate the following:

a: $x =$
b: $n =$
c: *p-hat* $=$

Exercise 9.15.7 *(Solution on p. 382.)*
Calculate σ_x. Make sure to show how you set up the formula.

Exercise 9.15.8 *(Solution on p. 382.)*
State the distribution to use for the hypothesis test.

Exercise 9.15.9
Sketch a graph of the situation. Label the horizontal axis. Mark the hypothesized mean and the sample proportion, p-hat. Shade the area corresponding to the p-value.

[15] This content is available online at <http://cnx.org/content/m17003/1.8/>.

Exercise 9.15.10 *(Solution on p. 382.)*

Find the p-value

Exercise 9.15.11 *(Solution on p. 382.)*

At a pre-conceived $\alpha = 0.05$, what is your:

a. Decision:
b. Reason for the decision:
c. Conclusion (write out in a complete sentence):

9.15.4 Discusion Question

Exercise 9.15.12

Does it appear that the proportion of people in that town with depression or a depressive illness is lower than general adult American population? Why or why not?

9.16 Homework[16]

Exercise 9.16.1 *(Solution on p. 382.)*

Some of the statements below refer to the null hypothesis, some to the alternate hypothesis.

State the null hypothesis, H_o, and the alternative hypothesis, H_a, in terms of the appropriate parameter (μ or p).

- **a.** Americans work an average of 34 years before retiring.
- **b.** At most 60% of Americans vote in presidential elections.
- **c.** The average starting salary for San Jose State University graduates is at least $100,000 per year.
- **d.** 29% of high school seniors get drunk each month.
- **e.** Fewer than 5% of adults ride the bus to work in Los Angeles.
- **f.** The average number of cars a person owns in her lifetime is not more than 10.
- **g.** About half of Americans prefer to live away from cities, given the choice.
- **h.** Europeans have an average paid vacation each year of six weeks.
- **i.** The chance of developing breast cancer is under 11% for women.
- **j.** Private universities cost, on average, more than $20,000 per year for tuition.

Exercise 9.16.2 *(Solution on p. 382.)*

For (a) - (j) above, state the Type I and Type II errors in complete sentences.

Exercise 9.16.3

For (a) - (j) above, in complete sentences:

- **a.** State a consequence of committing a Type I error.
- **b.** State a consequence of committing a Type II error.

DIRECTIONS: For each of the word problems, use a solution sheet to do the hypothesis test. The solution sheet is found in the Appendix. Please feel free to make copies of it. For the online version of the book, it is suggested that you copy the .doc or the .pdf files.

NOTE: If you are using a student-t distribution for a homework problem below, you may assume that the underlying population is normally distributed. (In general, you must first prove that assumption, though.)

Exercise 9.16.4

A particular brand of tires claims that its deluxe tire averages at least 50,000 miles before it needs to be replaced. From past studies of this tire, the standard deviation is known to be 8000. A survey of owners of that tire design is conducted. From the 28 tires surveyed, the average lifespan was 46,500 miles with a standard deviation of 9800 miles. Do the data support the claim at the 5% level?

Exercise 9.16.5 *(Solution on p. 382.)*

From generation to generation, the average age when smokers first start to smoke varies. However, the standard deviation of that age remains constant of around 2.1 years. A survey of 40 smokers of this generation was done to see if the average starting age is at least 19. The sample average was 18.1 with a sample standard deviation of 1.3. Do the data support the claim at the 5% level?

[16]This content is available online at <http://cnx.org/content/m17001/1.10/>.

Exercise 9.16.6

The cost of a daily newspaper varies from city to city. However, the variation among prices remains steady with a standard deviation of 6¢. A study was done to test the claim that the average cost of a daily newspaper is 35¢. Twelve costs yield an average cost of 30¢ with a standard deviation of 4¢. Do the data support the claim at the 1% level?

Exercise 9.16.7 ***(Solution on p. 382.)***

An article in the **San Jose Mercury News** stated that students in the California state university system take an average of 4.5 years to finish their undergraduate degrees. Suppose you believe that the average time is longer. You conduct a survey of 49 students and obtain a sample mean of 5.1 with a sample standard deviation of 1.2. Do the data support your claim at the 1% level?

Exercise 9.16.8

The average number of sick days an employee takes per year is believed to be about 10. Members of a personnel department do not believe this figure. They randomly survey 8 employees. The number of sick days they took for the past year are as follows: 12; 4; 15; 3; 11; 8; 6; 8. Let x = the number of sick days they took for the past year. Should the personnel team believe that the average number is about 10?

Exercise 9.16.9 ***(Solution on p. 382.)***

In 1955, **Life Magazine** reported that the 25 year-old mother of three worked [on average] an 80 hour week. Recently, many groups have been studying whether or not the women's movement has, in fact, resulted in an increase in the average work week for women (combining employment and at-home work). Suppose a study was done to determine if the average work week has increased. 81 women were surveyed with the following results. The sample average was 83; the sample standard deviation was 10. Does it appear that the average work week has increased for women at the 5% level?

Exercise 9.16.10

Your statistics instructor claims that 60 percent of the students who take her Elementary Statistics class go through life feeling more enriched. For some reason that she can't quite figure out, most people don't believe her. You decide to check this out on your own. You randomly survey 64 of her past Elementary Statistics students and find that 34 feel more enriched as a result of her class. Now, what do you think?

Exercise 9.16.11 ***(Solution on p. 382.)***

A Nissan Motor Corporation advertisement read, "The average man's I.Q. is 107. The average brown trout's I.Q. is 4. So why can't man catch brown trout?" Suppose you believe that the average brown trout's I.Q. is greater than 4. You catch 12 brown trout. A fish psychologist determines the I.Q.s as follows: 5; 4; 7; 3; 6; 4; 5; 3; 6; 3; 8; 5. Conduct a hypothesis test of your belief.

Exercise 9.16.12

Refer to the previous problem. Conduct a hypothesis test to see if your decision and conclusion would change if your belief were that the average brown trout's I.Q. is **not** 4.

Exercise 9.16.13 ***(Solution on p. 383.)***

According to an article in **Newsweek**, the natural ratio of girls to boys is 100:105. In China, the birth ratio is 100: 114 (46.7% girls). Suppose you don't believe the reported figures of the percent of girls born in China. You conduct a study. In this study, you count the number of girls and boys born in 150 randomly chosen recent births. There are 60 girls and 90 boys born of the 150. Based on your study, do you believe that the percent of girls born in China is 46.7?

Exercise 9.16.14

A poll done for **Newsweek** found that 13% of Americans have seen or sensed the presence of an angel. A contingent doubts that the percent is really that high. It conducts its own survey. Out of 76 Americans surveyed, only 2 had seen or sensed the presence of an angel. As a result of the contingent's survey, would you agree with the **Newsweek** poll? In complete sentences, also give three reasons why the two polls might give different results.

Exercise 9.16.15 *(Solution on p. 383.)*

The average work week for engineers in a start-up company is believed to be about 60 hours. A newly hired engineer hopes that it's shorter. She asks 10 engineering friends in start-ups for the lengths of their average work weeks. Based on the results that follow, should she count on the average work week to be shorter than 60 hours?

Data (length of average work week): 70; 45; 55; 60; 65; 55; 55; 60; 50; 55.

Exercise 9.16.16

Use the "Lap time" data for Lap 4 (see Table of Contents) to test the claim that Terri finishes Lap 4 on average in less than 129 seconds. Use all twenty races given.

Exercise 9.16.17

Use the "Initial Public Offering" data (see Table of Contents) to test the claim that the average offer price was $18 per share. Do not use all the data. Use your random number generator to randomly survey 15 prices.

NOTE: The following questions were written by past students. They are excellent problems!

Exercise 9.16.18

18. "Asian Family Reunion" by Chau Nguyen

```
    Every two years it comes around
We all get together from different towns.
In my honest opinion
It's not a typical family reunion
Not forty, or fifty, or sixty,
But how about seventy companions!
The kids would play, scream, and shout
One minute they're happy, another they'll pout.
The teenagers would look, stare, and compare
From how they look to what they wear.
The men would chat about their business
That they make more, but never less.
Money is always their subject
And there's always talk of more new projects.
The women get tired from all of the chats
They head to the kitchen to set out the mats.
Some would sit and some would stand
Eating and talking with plates in their hands.
Then come the games and the songs
And suddenly, everyone gets along!
With all that laughter, it's sad to say
That it always ends in the same old way.
They hug and kiss and say "good-bye"
And then they all begin to cry!
I say that 60 percent shed their tears
But my mom counted 35 people this year.
She said that boys and men will always have their pride,
So we won't ever see them cry.
I myself don't think she's correct,
```

```
So could you please try this problem to see if you object?
```

Exercise 9.16.19 *(Solution on p. 383.)*

"The Problem with Angels" by Cyndy Dowling

```
    Although this problem is wholly mine,
The catalyst came from the magazine, Time.
On the magazine cover I did find
The realm of angels tickling my mind.


    Inside, 69% I found to be
In angels, Americans do believe.


    Then, it was time to rise to the task,
Ninety-five high school and college students I did ask.
Viewing all as one group,
Random sampling to get the scoop.


    So, I asked each to be true,
"Do you believe in angels?"  Tell me, do!


    Hypothesizing at the start,
Totally believing in my heart
That the proportion who said yes
Would be equal on this test.


    Lo and behold, seventy-three did arrive,
Out of the sample of ninety-five.
Now your job has just begun,
Solve this problem and have some fun.
```

Exercise 9.16.20

"Blowing Bubbles" by Sondra Prull

```
    Studying stats just made me tense,
I had to find some sane defense.
Some light and lifting simple play
To float my math anxiety away.


    Blowing bubbles lifts me high
Takes my troubles to the sky.
POIK! They're gone, with all my stress
Bubble therapy is the best.


    The label said each time I blew
The average number of bubbles would be at least 22.
I blew and blew and this I found
From 64 blows, they all are round!
```

```
    But the number of bubbles in 64 blows
Varied widely, this I know.
20 per blow became the mean
They deviated by 6, and not 16.


    From counting bubbles, I sure did relax
But now I give to you your task.
Was 22 a reasonable guess?
Find the answer and pass this test!
```

Exercise 9.16.21 *(Solution on p. 383.)*

21. "Dalmatian Darnation" by Kathy Sparling

```
    A greedy dog breeder named Spreckles
Bred puppies with numerous freckles
The Dalmatians he sought
Possessed spot upon spot
The more spots, he thought, the more shekels.


    His competitors did not agree
That freckles would increase the fee.
They said, ‘‘Spots are quite nice
But they don’t affect price;
One should breed for improved pedigree.’’


    The breeders decided to prove
This strategy was a wrong move.
Breeding only for spots
Would wreak havoc, they thought.
His theory they want to disprove.


    They proposed a contest to Spreckles
Comparing dog prices to freckles.
In records they looked up
One hundred one pups:
Dalmatians that fetched the most shekels.


    They asked Mr. Spreckles to name
An average spot count he’d claim
To bring in big bucks.
Said Spreckles, ‘‘Well, shucks,
It’s for one hundred one that I aim.’’


    Said an amateur statistician
Who wanted to help with this mission.
‘‘Twenty-one for the sample
Standard deviation’s ample:
```

```
    They examined one hundred and one
Dalmatians that fetched a good sum.
They counted each spot,
Mark, freckle and dot
And tallied up every one.


    Instead of one hundred one spots
They averaged ninety six dots
Can they muzzle Spreckles'
Obsession with freckles
Based on all the dog data they've got?
```

Exercise 9.16.22
"Macaroni and Cheese, please!!" by Nedda Misherghi and Rachelle Hall

As a poor starving student I don't have much money to spend for even the bare necessities. So my favorite and main staple food is macaroni and cheese. It's high in taste and low in cost and nutritional value.

One day, as I sat down to determine the meaning of life, I got a serious craving for this, oh, so important, food of my life. So I went down the street to Greatway to get a box of macaroni and cheese, but it was SO expensive! $2.02 !!! Can you believe it? It made me stop and think. The world is changing fast. I had thought that the average cost of a box (the normal size, not some super-gigantic-family-value-pack) was at most $1, but now I wasn't so sure. However, I was determined to find out. I went to 53 of the closest grocery stores and surveyed the prices of macaroni and cheese. Here are the data I wrote in my notebook:

Price per box of Mac and Cheese:

- 5 stores @ $2.02
- 15 stores @ $0.25
- 3 stores @ $1.29
- 6 stores @ $0.35
- 4 stores @ $2.27
- 7 stores @ $1.50
- 5 stores @ $1.89
- 8 stores @ 0.75.

I could see that the costs varied but I had to sit down to figure out whether or not I was right. If it does turn out that this mouth-watering dish is at most $1, then I'll throw a big cheesy party in our next statistics lab, with enough macaroni and cheese for just me. (After all, as a poor starving student I can't be expected to feed our class of animals!)

Exercise 9.16.23 ***(Solution on p. 383.)***
"William Shakespeare: The Tragedy of Hamlet, Prince of Denmark" by Jacqueline Ghodsi

THE CHARACTERS (in order of appearance):

- HAMLET, Prince of Denmark and student of Statistics
- POLONIUS, Hamlet's tutor
- HOROTIO, friend to Hamlet and fellow student

Scene: The great library of the castle, in which Hamlet does his lessons

Act I

(The day is fair, but the face of Hamlet is clouded. He paces the large room. His tutor, Polonius, is reprimanding Hamlet regarding the latter's recent experience. Horatio is seated at the large table at right stage.)

POLONIUS: My Lord, how cans't thou admit that thou hast seen a ghost! It is but a figment of your imagination!

HAMLET: I beg to differ; I know of a certainty that five-and-seventy in one hundred of us, condemned to the whips and scorns of time as we are, have gazed upon a spirit of health, or goblin damn'd, be their intents wicked or charitable.

POLONIUS If thou doest insist upon thy wretched vision then let me invest your time; be true to thy work and speak to me through the reason of the null and alternate hypotheses. (He turns to Horatio.) Did not Hamlet himself say, "What piece of work is man, how noble in reason, how infinite in faculties? Then let not this foolishness persist. Go, Horatio, make a survey of three-and-sixty and discover what the true proportion be. For my part, I will never succumb to this fantasy, but deem man to be devoid of all reason should thy proposal of at least five-and-seventy in one hundred hold true.

HORATIO (to Hamlet): What should we do, my Lord?

HAMLET: Go to thy purpose, Horatio.

HORATIO: To what end, my Lord?

HAMLET: That you must teach me. But let me conjure you by the rights of our fellowship, by the consonance of our youth, but the obligation of our ever-preserved love, be even and direct with me, whether I am right or no.

(Horatio exits, followed by Polonius, leaving Hamlet to ponder alone.)

Act II

(The next day, Hamlet awaits anxiously the presence of his friend, Horatio. Polonius enters and places some books upon the table just a moment before Horatio enters.)

POLONIUS: So, Horatio, what is it thou didst reveal through thy deliberations?

HORATIO: In a random survey, for which purpose thou thyself sent me forth, I did discover that one-and-forty believe fervently that the spirits of the dead walk with us. Before my God, I might not this believe, without the sensible and true avouch of mine own eyes.

POLONIUS: Give thine own thoughts no tongue, Horatio. (Polonius turns to Hamlet.) But look to't I charge you, my Lord. Come Horatio, let us go together, for this is not our test. (Horatio and Polonius leave together.)

HAMLET: To reject, or not reject, that is the question: whether 'tis nobler in the mind to suffer the slings and arrows of outrageous statistics, or to take arms against a sea of data, and, by opposing, end them. (Hamlet resignedly attends to his task.)

(Curtain falls)

Exercise 9.16.24
"Untitled" by Stephen Chen

I've often wondered how software is released and sold to the public. Ironically, I work for a company that sells products with known problems. Unfortunately, most of the problems are difficult to create, which makes them difficult to fix. I usually use the test program X, which tests the product, to try to create a specific problem. When the test program is run to make an error occur, the likelihood of generating an error is 1%.

So, armed with this knowledge, I wrote a new test program Y that will generate the same error that test program X creates, but more often. To find out if my test program is better than the original, so that I can convince the management that I'm right, I ran my test program to find out how often I can generate the same error. When I ran my test program 50 times, I generated the error twice. While this may not seem much better, I think that I can convince the management to use my test program instead of the original test program. Am I right?

Exercise 9.16.25 ***(Solution on p. 383.)***

Japanese Girls' Names

by Kumi Furuichi

It used to be very typical for Japanese girls' names to end with "ko." (The trend might have started around my grandmothers' generation and its peak might have been around my mother's generation.) "Ko" means "child" in Chinese character. Parents would name their daughters with "ko" attaching to other Chinese characters which have meanings that they want their daughters to become, such as Sachiko – a happy child, Yoshiko – a good child, Yasuko – a healthy child, and so on.

However, I noticed recently that only two out of nine of my Japanese girlfriends at this school have names which end with "ko." More and more, parents seem to have become creative, modernized, and, sometimes, westernized in naming their children.

I have a feeling that, while 70 percent or more of my mother's generation would have names with "ko" at the end, the proportion has dropped among my peers. I wrote down all my Japanese friends', ex-classmates', co-workers, and acquaintances' names that I could remember. Below are the names. (Some are repeats.) Test to see if the proportion has dropped for this generation.

Ai, Akemi, Akiko, Ayumi, Chiaki, Chie, Eiko, Eri, Eriko, Fumiko, Harumi, Hitomi, Hiroko, Hiroko, Hidemi, Hisako, Hinako, Izumi, Izumi, Junko, Junko, Kana, Kanako, Kanayo, Kayo, Kayoko, Kazumi, Keiko, Keiko, Kei, Kumi, Kumiko, Kyoko, Kyoko, Madoka, Maho, Mai, Maiko, Maki, Miki, Miki, Mikiko, Mina, Minako, Miyako, Momoko, Nana, Naoko, Naoko, Naoko, Noriko, Rieko, Rika, Rika, Rumiko, Rei, Reiko, Reiko, Sachiko, Sachiko, Sachiyo, Saki, Sayaka, Sayoko, Sayuri, Seiko, Shiho, Shizuka, Sumiko, Takako, Takako, Tomoe, Tomoe, Tomoko, Touko, Yasuko, Yasuko, Yasuyo, Yoko, Yoko, Yoko, Yoshiko, Yoshiko, Yoshiko, Yuka, Yuki, Yuki, Yukiko, Yuko, Yuko.

Exercise 9.16.26

Phillip's Wish by Suzanne Osorio

```
    My nephew likes to play
Chasing the girls makes his day.
He asked his mother
If it is okay
To get his ear pierced.
She said, ''No way!''
To poke a hole through your ear,
Is not what I want for you, dear.
He argued his point quite well,
```

```
Says even my macho pal,  Mel,
Has gotten this done.
It's all just for fun.
C'mon please, mom, please, what the hell.
Again Phillip complained to his mother,
Saying half his friends (including their brothers)
Are piercing their ears
And they have no fears
He wants to be like the others.
She said, ''I think it's much less.
We must do a hypothesis test.
And if you are right,
I won't put up a fight.
But, if not, then my case will rest.''
We proceeded to call fifty guys
To see whose prediction would fly.
Nineteen of the fifty
Said piercing was nifty
And earrings they'd occasionally buy.
Then there's the other thirty-one,
Who said they'd never have this done.
So now this poem's finished.
Will his hopes be diminished,
Or will my nephew have his fun?
```

Exercise 9.16.27 *(Solution on p. 383.)*

The Craven by Mark Salangsang

```
    Once upon a morning dreary
In stats class I was weak and weary.
Pondering over last night's homework
Whose answers were now on the board
This I did and nothing more.

    While I nodded nearly napping
Suddenly, there came a tapping.
As someone gently rapping,
Rapping my head as I snore.
Quoth the teacher, ''Sleep no more.''

    ''In every class you fall asleep,''
The teacher said, his voice was deep.
''So a tally I've begun to keep
Of every class you nap and snore.
The percentage being forty-four.''

    ''My dear teacher I must confess,
While sleeping is what I do best.
The percentage, I think, must be less,
A percentage less than forty-four.''
This I said and nothing more.
```

```
    ‘‘We’ll see,’’ he said and walked away,
And fifty classes from that day
He counted till the month of May
The classes in which I napped and snored.
The number he found was twenty-four.


    At a significance level of 0.05,
Please tell me am I still alive?
Or did my grade just take a dive
Plunging down beneath the floor?
Upon thee I hereby implore.
```

Exercise 9.16.28

Toastmasters International cites a February 2001 report by Gallop Poll that 40% of Americans fear public speaking. A student believes that less than 40% of students at her school fear public speaking. She randomly surveys 361 schoolmates and finds that 135 report they fear public speaking. Conduct a hypothesis test to determine if the percent at her school is less than 40%. (*Source: http://toastmasters.org/artisan/detail.asp?CategoryID=1&SubCategoryID=10&ArticleID=429&Page=1*[17])

Exercise 9.16.29 ***(Solution on p. 383.)***

In 2004, 68% of online courses taught at community colleges nationwide were taught by full-time faculty. To test if 68% also represents California's percent for full-time faculty teaching the online classes, Long Beach City College (LBCC), CA, was randomly selected for comparison. In 2004, 34 of the 44 online courses LBCC offered were taught by full-time faculty. Conduct a hypothesis test to determine if 68% represents CA. NOTE: For a true test, use more CA community colleges. (Sources: **Growing by Degrees** by Allen and Seaman; Amit Schitai, Director of Instructional Technology and Distance Learning, LBCC).

NOTE: For a true test, use more CA community colleges.

Exercise 9.16.30

According to an article in **The New York Times** (5/12/2004), 19.3% of New York City adults smoked in 2003. Suppose that a survey is conducted to determine this year's rate. Twelve out of 70 randomly chosen N.Y. City residents reply that they smoke. Conduct a hypothesis test to determine is the rate is still 19.3%.

Exercise 9.16.31 ***(Solution on p. 384.)***

The average age of De Anza College students in Winter 2006 term was 26.6 years old. An instructor thinks the average age for online students is older than 26.6. She randomly surveys 56 online students and finds that the sample average is 29.4 with a standard deviation of 2.1. Conduct a hypothesis test. (*Source: http://research.fhda.edu/factbook/DAdemofs/Fact_sheet_da_2006w.pdf*[18])

Exercise 9.16.32

In 2004, registered nurses earned an average annual salary of $52,330. A survey was conducted of 41 California nursed to determine if the annual salary is higher than $52,330 for California nurses. The sample average was $61,121 with a sample standard deviation of $7,489. Conduct a hypothesis test. (*Source: http://stats.bls.gov/oco/ocos083.htm#earnings*[19])

[17] http://toastmasters.org/artisan/detail.asp?CategoryID=1&SubCategoryID=10&ArticleID=429&Page=1
[18] http://research.fhda.edu/factbook/DAdemofs/Fact_sheet_da_2006w.pdf
[19] http://stats.bls.gov/oco/ocos083.htm#earnings

Exercise 9.16.33 *(Solution on p. 384.)*

La Leche League International reports that the average age of weaning a child from breastfeeding is age 4 to 5 worldwide. In America, most nursing mothers wean their children much earlier. Suppose a random survey is conducted of 21 U.S. mothers who recently weaned their children. The average weaning age was 9 months (3/4 year) with a standard deviation of 4 months. Conduct a hypothesis test to determine is the average weaning age in the U.S. is less than 4 years old. (*Source: http://www.lalecheleague.org/Law/BAFeb01.html*[20])

9.16.1 Try these multiple choice questions.

Exercise 9.16.34 *(Solution on p. 384.)*

When a new drug is created, the pharmaceutical company must subject it to testing before receiving the necessary permission from the Food and Drug Administration (FDA) to market the drug. Suppose the null hypothesis is "the drug is unsafe." What is the Type II Error?

A. To claim the drug is safe when in, fact, it is unsafe
B. To claim the drug is unsafe when, in fact, it is safe.
C. To claim the drug is safe when, in fact, it is safe.
D. To claim the drug is unsafe when, in fact, it is unsafe

The next two questions refer to the following information: Over the past few decades, public health officials have examined the link between weight concerns and teen girls smoking. Researchers surveyed a group of 273 randomly selected teen girls living in Massachusetts (between 12 and 15 years old). After four years the girls were surveyed again. Sixty-three (63) said they smoked to stay thin. Is there good evidence that more than thirty percent of the teen girls smoke to stay thin?

Exercise 9.16.35 *(Solution on p. 384.)*

The alternate hypothesis is

A. $p < 0.30$
B. $p \leq 0.30$
C. $p \geq 0.30$
D. $p > 0.30$

Exercise 9.16.36 *(Solution on p. 384.)*

After conducting the test, your decision and conclusion are

A. Reject H_o: More than 30% of teen girls smoke to stay thin.
B. Do not reject H_o: Less than 30% of teen girls smoke to stay thin.
C. Do not reject H_o: At most 30% of teen girls smoke to stay thin.
D. Reject H_o: Less than 30% of teen girls smoke to stay thin.

The next three questions refer to the following information: A statistics instructor believes that fewer than 20% of Evergreen Valley College (EVC) students attended the opening night midnight showing of the latest Harry Potter movie. She surveys 84 of her students and finds that 11 of attended the midnight showing.

Exercise 9.16.37 *(Solution on p. 384.)*

An appropriate alternative hypothesis is

[20] http://www.lalecheleague.org/Law/BAFeb01.html

A. $p = 0.20$
B. $p > 0.20$
C. $p < 0.20$
D. $p \leq 0.20$

Exercise 9.16.38 *(Solution on p. 384.)*

At a 1% level of significance, an appropriate conclusion is:

A. The percent of EVC students who attended the midnight showing of Harry Potter is at least 20%.
B. The percent of EVC students who attended the midnight showing of Harry Potter is more than 20%.
C. The percent of EVC students who attended the midnight showing of Harry Potter is less than 20%.
D. There is not enough information to make a decision.

Exercise 9.16.39 *(Solution on p. 384.)*

The Type I error is believing that the percent of EVC students who attended is:

A. at least 20%, when in fact, it is less than 20%.
B. 20%, when in fact, it is 20%.
C. less than 20%, when in fact, it is at least 20%.
D. less than 20%, when in fact, it is less than 20%.

The next two questions refer to the following information:

It is believed that Lake Tahoe Community College (LTCC) Intermediate Algebra students get less than 7 hours of sleep per night, on average. A survey of 22 LTCC Intermediate Algebra students generated an average of 7.24 hours with a standard deviation of 1.93 hours. At a level of significance of 5%, do LTCC Intermediate Algebra students get less than 7 hours of sleep per night, on average?

Exercise 9.16.40 *(Solution on p. 384.)*

The distribution to be used for this test is $\overline{X} \sim$

A. $N\left(7.24, \frac{1.93}{\sqrt{22}}\right)$
B. $N\left(7.24, 1.93\right)$
C. t_{22}
D. t_{21}

Exercise 9.16.41 *(Solution on p. 384.)*

The Type II error is "I believe that the average number of hours of sleep LTCC students get per night

A. is less than 7 hours when, in fact, it is at least 7 hours."
B. is less than 7 hours when, in fact, it is less than 7 hours."
C. is at least 7 hours when, in fact, it is at least 7 hours."
D. is at least 7 hours when, in fact, it is less than 7 hours."

The next three questions refer to the following information: An organization in 1995 reported that teenagers spent an average of 4.5 hours per week on the telephone. The organization thinks that, in 2007, the average is higher. Fifteen (15) randomly chosen teenagers were asked how many hours per week they spend on the telephone. The sample mean was 4.75 hours with a sample standard deviation of 2.0.

Exercise 9.16.42 *(Solution on p. 384.)*

The null and alternate hypotheses are:

A. $H_o : \overline{x} = 4.5, H_a : \overline{x} > 4.5$
B. $H_o : \mu \geq 4.5$ $H_a : \mu < 4.5$
C. $H_o : \mu = 4.75$ $H_{a:}\mu > 4.75$
D. $H_o : \mu = 4.5$ $H_a : \mu > 4.5$

Exercise 9.16.43 ***(Solution on p. 384.)***

At a significance level of $a = 0.05$, the correct conclusion is:

A. The average in 2007 is higher than it was in 1995.
B. The average in 1995 is higher than in 2007.
C. The average is still about the same as it was in 1995.
D. The test is inconclusive.

Exercise 9.16.44 ***(Solution on p. 384.)***

The Type I error is:

A. To conclude the average hours per week in 2007 is higher than in 1995, when in fact, it is higher.
B. To conclude the average hours per week in 2007 is higher than in 1995, when in fact, it is the same.
C. To conclude the average hours per week in 2007 is the same as in 1995, when in fact, it is higher.
D. To conclude the average hours per week in 2007 is no higher than in 1995, when in fact, it is not higher.

9.17 Review[21]

Exercise 9.17.1 *(Solution on p. 384.)*

1. Rebecca and Matt are 14 year old twins. Matt's height is 2 standard deviations below the mean for 14 year old boys' height. Rebecca's height is 0.10 standard deviations above the mean for 14 year old girls' height. Interpret this.

A. Matt is 2.1 inches shorter than Rebecca
B. Rebecca is very tall compared to other 14 year old girls.
C. Rebecca is taller than Matt.
D. Matt is shorter than the average 14 year old boy.

2. Construct a histogram of the IPO data (see Table of Contents, 14. Appendix, Data Sets). Use 5 intervals.

The next six questions refer to the following information: Ninety homeowners were asked the number of estimates they obtained before having their homes fumigated. X = the number of estimates.

X	**Rel. Freq.**	**Cumulative Rel. Freq.**
1	0.3	
2	0.2	
4	0.4	
5	0.1	

Table 9.5

3. Calculate the frequencies.

4. Complete the cumulative relative frequency column. What percent of the estimates fell at or below 4?

Exercise 9.17.2 *(Solution on p. 384.)*

5. Calculate the sample mean (a) and sample standard deviation (b).

Exercise 9.17.3 *(Solution on p. 384.)*

6. Calculate the median, M, the first quartile, Q1, the third quartile, Q3.

Exercise 9.17.4 *(Solution on p. 384.)*

7. The middle 50% of the data are between _____ and _____.

8. Construct a boxplot of the data.

The next three questions refer to the following table: Seventy 5th and 6th graders were asked their favorite dinner.

	Pizza	**Hamburgers**	**Spaghetti**	**Fried shrimp**
5th grader	15	6	9	0
6th grader	15	7	10	8

Table 9.6

Exercise 9.17.5 *(Solution on p. 384.)*

9. Find the probability that one randomly chosen child is in the 6th grade and prefers fried shrimp.

[21] This content is available online at <http://cnx.org/content/m17013/1.9/>.

A. $\frac{32}{70}$
B. $\frac{8}{32}$
C. $\frac{8}{8}$
D. $\frac{8}{70}$

Exercise 9.17.6 ***(Solution on p. 385.)***
10. Find the probability that a child does not prefer pizza.

A. $\frac{30}{70}$
B. $\frac{30}{40}$
C. $\frac{40}{70}$
D. 1

Exercise 9.17.7 ***(Solution on p. 385.)***
11. Find the probability a child is in the 5th grade given that the child prefers spaghetti.

A. $\frac{9}{19}$
B. $\frac{9}{70}$
C. $\frac{9}{30}$
D. $\frac{19}{70}$

Exercise 9.17.8 ***(Solution on p. 385.)***
12. A sample of convenience is a random sample.

A. true
B. false

Exercise 9.17.9 ***(Solution on p. 385.)***
13. A statistic is a number that is a property of the population.

A. true
B. false

Exercise 9.17.10 ***(Solution on p. 385.)***
14. You should always throw out any data that are outliers.

A. true
B. false

Exercise 9.17.11 ***(Solution on p. 385.)***
15. Lee bakes pies for a little restaurant in Felton. She generally bakes 20 pies in a day, on the average.

a. Define the Random Variable X.
b. State the distribution for X.
c. Find the probability that Lee bakes more than 25 pies in any given day.

Exercise 9.17.12 ***(Solution on p. 385.)***
16. Six different brands of Italian salad dressing were randomly selected at a supermarket. The grams of fat per serving are 7, 7, 9, 6, 8, 5. Assume that the underlying distribution is normal. Calculate a 95% confidence interval for the population average grams of fat per serving of Italian salad dressing sold in supermarkets.

Exercise 9.17.13 ***(Solution on p. 385.)***
17. Given: uniform, exponential, normal distributions. Match each to a statement below.

a. mean = median $\neq$ mode
b. mean $>$ median $>$ mode
c. mean = median = mode

9.18 Lab: Hypothesis Testing of a Single Mean and Single Proportion[22]

Class Time:

Names:

9.18.1 Student Learning Outcomes:

- The student will select the appropriate distributions to use in each case.
- The student will conduct hypothesis tests and interpret the results.

9.18.2 Television Survey

In a recent survey, it was stated that Americans watch television on average four hours per day. Assume that $\sigma = 2$. Using your class as the sample, conduct a hypothesis test to determine if the average for students at your school is lower.

1. H_o:
2. H_a:
3. In words, define the random variable. __________ =
4. The distribution to use for the test is:
5. Determine the test statistic using your data.
6. Draw a graph and label it appropriately.Shade the actual level of significance.
   a. Graph:

[22]This content is available online at <http://cnx.org/content/m17007/1.9/>.

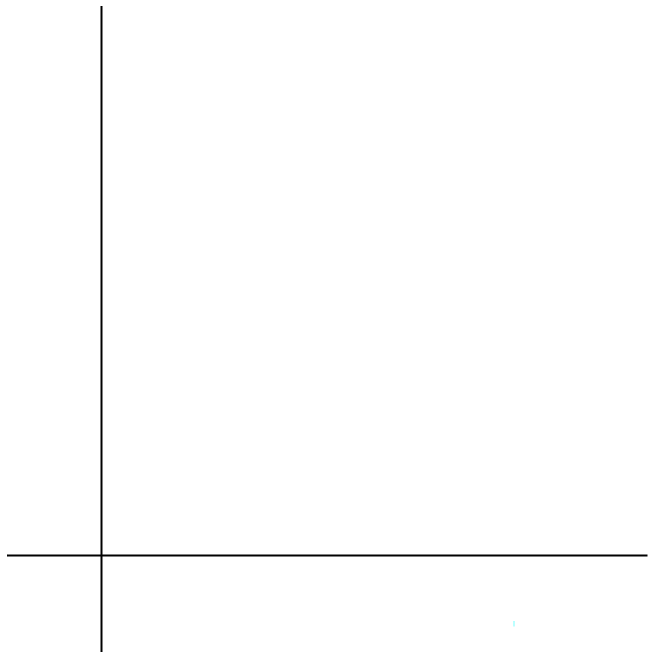


Figure 9.9

b. Determine the p-value:

7. Do you or do you not reject the null hypothesis? Why?
8. Write a clear conclusion using a complete sentence.

9.18.3 Language Survey

According to the 2000 Census, about 39.5% of Californians and 17.9% of all Americans speak a language other than English at home. Using your class as the sample, conduct a hypothesis test to determine if the percent of the students at your school that speak a language other than English at home is different from 39.5%.

1. H_o:
2. H_a:
3. In words, define the random variable. __________ =
4. The distribution to use for the test is:
5. Determine the test statistic using your data.
6. Draw a graph and label it appropriately. Shade the actual level of significance.

a. Graph:

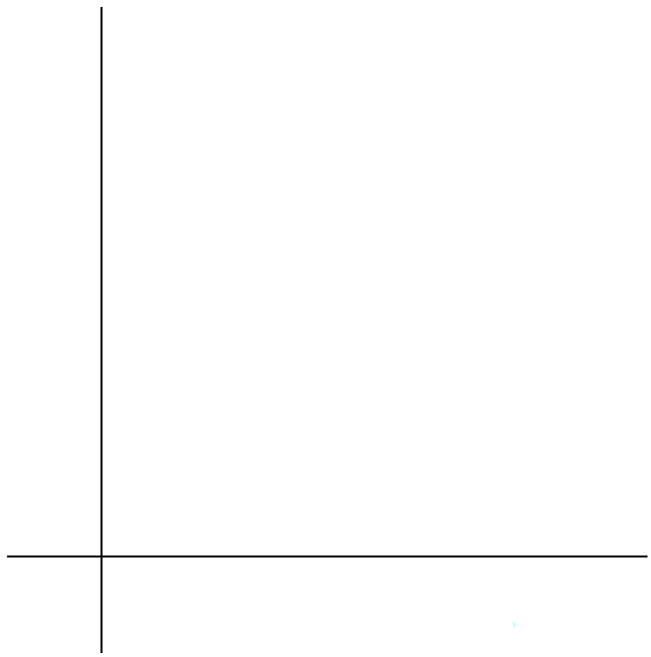


Figure 9.10

b. Determine the p-value:

7. Do you or do you not reject the null hypothesis? Why?
8. Write a clear conclusion using a complete sentence.

9.18.4 Jeans Survey

Suppose that young adults own an average of 3 pairs of jeans. Survey 8 people from your class to determine if the average is higher than 3.

1. H_0:
2. H_a:
3. In words, define the random variable. __________ =
4. The distribution to use for the test is:
5. Determine the test statistic using your data.
6. Draw a graph and label it appropriately. Shade the actual level of significance.

    a. Graph:

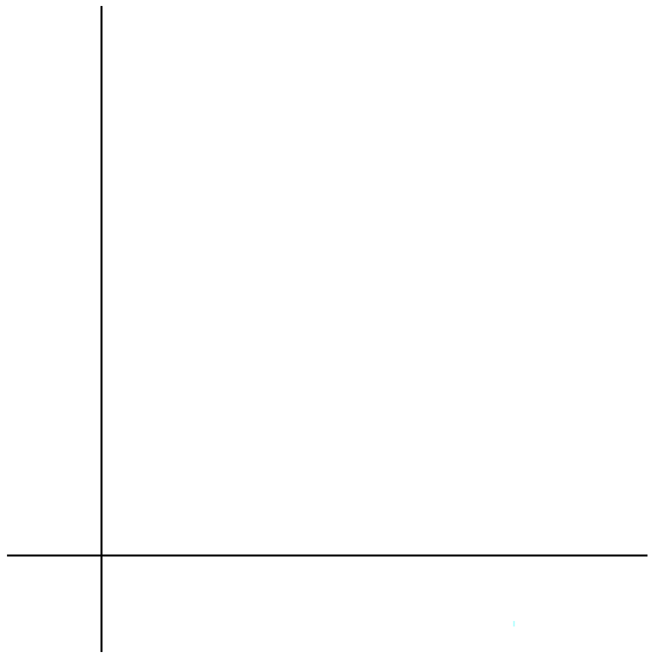


Figure 9.11

b. Determine the p-value:

7. Do you or do you not reject the null hypothesis? Why?
8. Write a clear conclusion using a complete sentence.

Solutions to Exercises in Chapter 9

Solution to Example 9.15, Problem 2 (p. 349)
$p' = \frac{x}{n}$ where x is the number of successes and n is the total number in the sample.

$x = 43$, $n = 150$

p′ = $\frac{43}{150}$
Solution to Example 9.15, Problem 3 (p. 349)
A success is having three cell phones in a household.
Solution to Example 9.15, Problem 4 (p. 349)
The level of significance is the preset α. Since α is not given, assume that $\alpha = 0.5$.
Solution to Example 9.15, Problem 5 (p. 349)
p-value = 0.7216
Solution to Example 9.15, Problem 6 (p. 349)
Assuming that α = 0.5, $\alpha < p\text{-}value$. The Decision is do not reject H_0 because there is not sufficient evidence to conclude that the proportion of households that have three cell phones is not 30%.

Solutions to Practice 1: Single Mean, Known Population Standard Deviation

Solution to Exercise 9.13.1 (p. 354)
Averages
Solution to Exercise 9.13.2 (p. 354)

- **a:** $H_o : \mu = 2.5$ (or, $H_o : \mu \leq 2.5$)
- **b:** $H_a : \mu > 2.5$

Solution to Exercise 9.13.3 (p. 354)
right-tailed
Solution to Exercise 9.13.4 (p. 354)
$\overline{X}$
Solution to Exercise 9.13.5 (p. 354)
The average time spent in jail for 26 first time convicted burglars
Solution to Exercise 9.13.6 (p. 354)
Yes, 1.5
Solution to Exercise 9.13.7 (p. 354)

- **a.** 3
- **b.** 1.5
- **c.** 1.8
- **d.** 26

Solution to Exercise 9.13.8 (p. 354)
σ
Solution to Exercise 9.13.9 (p. 354)
$\overline{X} \sim N\left(2.5, \frac{1.5}{\sqrt{26}}\right)$
Solution to Exercise 9.13.11 (p. 355)
0.0446
Solution to Exercise 9.13.12 (p. 355)

- **a.** Reject the null hypothesis

Solutions to Practice 2: Single Mean, Unknown Population Standard Deviation

Solution to Exercise 9.14.1 (p. 356)
averages
Solution to Exercise 9.14.2 (p. 356)

- **a.** $H_o : \mu = 15$
- **b.** $H_a : \mu \neq 15$

Solution to Exercise 9.14.3 (p. 356)
two-tailed
Solution to Exercise 9.14.4 (p. 356)
$\overline{X}$
Solution to Exercise 9.14.5 (p. 356)
the average time spent on death row
Solution to Exercise 9.14.6 (p. 356)
No
Solution to Exercise 9.14.7 (p. 356)

- **a.** 17.4
- **b.** s
- **c.** 75

Solution to Exercise 9.14.8 (p. 356)
$t-$test
Solution to Exercise 9.14.9 (p. 356)
t_{74}
Solution to Exercise 9.14.11 (p. 357)
0.0015
Solution to Exercise 9.14.12 (p. 357)

- **a.** Reject the null hypothesis

Solutions to Practice 3: Single Proportion

Solution to Exercise 9.15.1 (p. 358)
Proportions
Solution to Exercise 9.15.2 (p. 358)

- **a.** $H_o : p = 0.095$
- **b.** $H_a : P < 0.095$

Solution to Exercise 9.15.3 (p. 358)
left-tailed
Solution to Exercise 9.15.4 (p. 358)
P-hat
Solution to Exercise 9.15.5 (p. 358)
the proportion of people in that town suffering from depress. or a depr. illness
Solution to Exercise 9.15.6 (p. 358)

- **a.** 7
- **b.** 100
- **c.** 0.07

Solution to Exercise 9.15.7 (p. 358)
2.93
Solution to Exercise 9.15.8 (p. 358)
Normal
Solution to Exercise 9.15.10 (p. 359)
0.1969
Solution to Exercise 9.15.11 (p. 359)

a. Do not reject the null hypothesis

Solutions to Homework

Solution to Exercise 9.16.1 (p. 360)

a. $H_o : \mu = 34$; $H_a : \mu \neq 34$
c. $H_o : \mu \geq 100,000$; $H_a : \mu < 100,000$
d. $H_o : p = 0.29$; $H_a : p \neq 0.29$
g. $H_o : p = 0.50$; $H_a : p \neq 0.50$
i. $H_o : p \geq 0.11$; $H_a : p < 0.11$

Solution to Exercise 9.16.2 (p. 360)

a. Type I error: We believe the average is not 34 years, when it really is 34 years. Type II error: We believe the average is 34 years, when it is not really 34 years.
c. Type I error: We believe the average is less than $100,000, when it really is at least $100,000. Type II error: We believe the average is at least $100,000, when it is really less than $100,000.
d. Type I error: We believe that the proportion of h.s. seniors who get drunk each month is not 29%, when it really is 29%. Type II error: We believe that 29% of h.s. seniors get drunk each month, when the proportion is really not 29%.
i. Type I error: We believe the proportion is less than 11%, when it is really at least 11%. Type II error: WE believe the proportion is at least 11%, when it really is less than 11%.

Solution to Exercise 9.16.5 (p. 360)

e. $z = -2.71$
f. 0.0034
h. Decision: Reject null; Conclusion: $\mu < 19$
i. $(17.449, 18.757)$

Solution to Exercise 9.16.7 (p. 361)

e. 3.5
f. 0.0005
h. Decision: Reject null; Conclusion: $\mu > 4.5$
i. $(4.7553, 5.4447)$

Solution to Exercise 9.16.9 (p. 361)

e. 2.7
f. 0.0042
h. Decision: Reject Null
i. $(80.789, 85.211)$

Solution to Exercise 9.16.11 (p. 361)

d. t_{11}

- **e.** 1.96
- **f.** 0.0380
- **h.** Decision: Reject null when $a = 0.05$; do not reject null when $a = 0.01$
- **i.** $(3.8865, 5.9468)$

Solution to Exercise 9.16.13 (p. 361)

- **e.** -1.64
- **f.** 0.1000
- **h.** Decision: Do not reject null
- **i.** $(0.3216, 0.4784)$

Solution to Exercise 9.16.15 (p. 362)

- **d.** t_9
- **e.** -1.33
- **f.** 0.1086
- **h.** Decision: Do not reject null
- **i.** $(51.886, 62.114)$

Solution to Exercise 9.16.19 (p. 363)

- **e.** 1.65
- **f.** 0.0984
- **h.** Decision: Do not reject null
- **i.** $(0.6836, 0.8533)$

Solution to Exercise 9.16.21 (p. 364)

- **e.** -2.39
- **f.** 0.0093
- **h.** Decision: Reject null
- **i.** $(91.854, 100.15)$

Solution to Exercise 9.16.23 (p. 365)

- **e.** -1.82
- **f.** 0.0345
- **h.** Decision: Do not reject null
- **i.** $(0.5331, 0.7685)$

Solution to Exercise 9.16.25 (p. 367)

- **e.** $z = -2.99$
- **f.** 0.0014
- **h.** Decision: Reject null; Conclusion: $p < .70$
- **i.** $(0.4529, 0.6582)$

Solution to Exercise 9.16.27 (p. 368)

- **e.** 0.57
- **f.** 0.7156
- **h.** Decision: Do not reject null
- **i.** $(0.3415, 0.6185)$

Solution to Exercise 9.16.29 (p. 369)

- **e.** 1.32

f. 0.1873
h. Decision: Do not reject null
i. $(0.65, 0.90)$

Solution to Exercise 9.16.31 (p. 369)

e. 9.98
f. 0.0000
h. Decision: Reject null
i. $(28.8, 30.0)$

Solution to Exercise 9.16.33 (p. 370)

e. -44.7
f. 0.0000
h. Decision: Reject null
i. $(0.60, 0.90)$ - in years

Solution to Exercise 9.16.34 (p. 370)
B
Solution to Exercise 9.16.35 (p. 370)
D
Solution to Exercise 9.16.36 (p. 370)
C
Solution to Exercise 9.16.37 (p. 370)
C
Solution to Exercise 9.16.38 (p. 371)
A
Solution to Exercise 9.16.39 (p. 371)
C
Solution to Exercise 9.16.40 (p. 371)
D
Solution to Exercise 9.16.41 (p. 371)
D
Solution to Exercise 9.16.42 (p. 371)
D
Solution to Exercise 9.16.43 (p. 372)
C
Solution to Exercise 9.16.44 (p. 372)
B

Solutions to Review

Solution to Exercise 9.17.1 (p. 373)
D
Solution to Exercise 9.17.2 (p. 373)

a. 2.8
b. 1.48

Solution to Exercise 9.17.3 (p. 373)
$M = 3$; $Q1 = 1$; $Q3 = 4$
Solution to Exercise 9.17.4 (p. 373)
1 and 4

Solution to Exercise 9.17.5 (p. 373)
D
Solution to Exercise 9.17.6 (p. 374)
C
Solution to Exercise 9.17.7 (p. 374)
A
Solution to Exercise 9.17.8 (p. 374)
B
Solution to Exercise 9.17.9 (p. 374)
B
Solution to Exercise 9.17.10 (p. 374)
B
Solution to Exercise 9.17.11 (p. 374)

- **b.** $P(20)$
- **c.** 0.1122

Solution to Exercise 9.17.12 (p. 374)
CI: $(5.52, 8.48)$
Solution to Exercise 9.17.13 (p. 374)

- **a.** uniform
- **b.** exponential
- **c.** normal