Chapter 10

Hypothesis Testing: Two Means, Paired Data, Two Proportions

10.1 Hypothesis Testing: Two Population Means and Two Population Proportions[1]

10.1.1 Student Learning Objectives

By the end of this chapter, the student should be able to:

- Classify hypothesis tests by type.
- Conduct and interpret hypothesis tests for two population means, population standard deviations known.
- Conduct and interpret hypothesis tests for two population means, population standard deviations unknown.
- Conduct and interpret hypothesis tests for two population proportions.
- Conduct and interpret hypothesis tests for matched or paired samples.

10.1.2 Introduction

Studies often compare two groups. For example, researchers are interested in the effect aspirin has in preventing heart attacks. Over the last few years, newspapers and magazines have reported about various aspirin studies involving two groups. Typically, one group is given aspirin and the other group is given a placebo. Then, the heart attack rate is studied over several years.

There are other situations that deal with the comparison of two groups. For example, studies compare various diet and exercise programs. Politicians compare the proportion of individuals from different income brackets who might vote for them. Students are interested in whether SAT or GRE preparatory courses really help raise their scores.

In the previous chapter, you learned to conduct hypothesis tests on single means and single proportions. You will expand upon that in this chapter. You will compare two averages or two proportions to each other. The general procedure is still the same, just expanded.

[1] This content is available online at <http://cnx.org/content/m17029/1.6/>.

To compare two averages or two proportions, you work with two groups. The groups are classified either as **independent** or **matched pairs**. **Independent groups** mean that the two samples taken are independent, that is, sample values selected from one population are not related in any way to sample values selected from the other population. **Matched pairs** consist of two samples that are dependent. The parameter tested using matched pairs is the population mean. The parameters tested using independent groups are either population means or population proportions.

> NOTE: This chapter relies on either a calculator or a computer to calculate the degrees of freedom, the test statistics, and p-values. TI-83+ and TI-84 instructions are included as well as the the test statistic formulas. Because of technology, we do not need to separate two population means, independent groups, population variances unknown into large and small sample sizes.

This chapter deals with the following hypothesis tests:

Independent groups (samples are independent)

- Test of two population means.
- Test of two population proportions.

Matched or paired samples (samples are dependent)

- Becomes a test of one population mean.

10.2 Comparing Two Independent Population Means with Unknown Population Standard Deviations[2]

1. The two independent samples are simple random samples from two distinct populations.
2. Both populations are normally distributed with the population means and standard deviations unknown unless the sample sizes are greater than 30. In that case, the populations need not be normally distributed.

The comparison of two population means is very common. A difference between the two samples depends on both the means and the standard deviations. Very different means can occur by chance if there is great variation among the individual samples. In order to account for the variation, we take the difference of the sample means, $\overline{X_1}$ - $\overline{X_2}$, and divide by the standard error (shown below) in order to standardize the difference. The result is a t-score test statistic (shown below).

Because we do not know the population standard deviations, we estimate them using the two sample standard deviations from our independent samples. For the hypothesis test, we calculate the estimated standard deviation, or **standard error**, of **the difference in sample means**, $\overline{X_1}$ - $\overline{X_2}$.

The standard error is:

$$\sqrt{\frac{(S_1)^2}{n_1}+\frac{(S_2)^2}{n_2}} \qquad (10.1)$$

The test statistic (t-score) is calculated as follows:

T-score

$$\frac{(\overline{x_1}-\overline{x_2})-(\mu_1-\mu_2)}{\sqrt{\frac{(S_1)^2}{n_1}+\frac{(S_2)^2}{n_2}}} \qquad (10.2)$$

where:

[2] This content is available online at <http://cnx.org/content/m17025/1.13/>.

- s_1 and s_2, the sample standard deviations, are estimates of σ_1 and σ_2, respectively.
- σ_1 and σ_2 are the unknown population standard deviations.
- $\overline{x_1}$ and $\overline{x_2}$ are the sample means. μ_1 and μ_2 are the population means.

The **degrees of freedom (df)** is a somewhat complicated calculation. However, a computer or calculator calculates it easily. The dfs are not always a whole number. The test statistic calculated above is approximated by the Student-t distribution with dfs as follows:

Degrees of freedom

$$df = \frac{\left[\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}\right]^2}{\frac{1}{n_1-1}\cdot\left[\frac{(s_1)^2}{n_1}\right]^2 + \frac{1}{n_2-1}\cdot\left[\frac{(s_2)^2}{n_2}\right]^2} \tag{10.3}$$

When both sample sizes n_1 and n_2 are five or larger, the Student-t approximation is very good. Notice that the sample variances ${s_1}^2$ and ${s_2}^2$ are not pooled. (If the question comes up, do not pool the variances.)

NOTE: It is not necessary to compute this by hand. A calculator or computer easily computes it.

Example 10.1: Independent groups

The average amount of time boys and girls ages 7 through 11 spend playing sports each day is believed to be the same. An experiment is done, data is collected, resulting in the table below:

	Sample Size	Average Number of Hours Playing Sports Per Day	Sample Standard Deviation
Girls	9	2 hours	$\sqrt{0.75}$
Boys	16	3.2 hours	1.00

Table 10.1

Problem

Is there a difference in the average amount of time boys and girls ages 7 through 11 play sports each day? Test at the 5% level of significance.

Solution

The population standard deviations are not known. Let g be the subscript for girls and b be the subscript for boys. Then, μ_g is the population mean for girls and μ_b is the population mean for boys. This is a test of two **independent groups**, two population **means**.

Random variable: $\overline{X_g} - \overline{X_b}$ = difference in the average amount of time girls and boys play sports each day.

H_o: $\mu_g = \mu_b \left(\mu_g - \mu_b = 0\right)$

H_a: $\mu_g \neq \mu_b \left(\mu_g - \mu_b \neq 0\right)$

The words **"the same"** tell you H_o has an "=". Since there are no other words to indicate H_a, then assume **"is different."** This is a two-tailed test.

Distribution for the test: Use t_{df} where df is calculated using the df formula for independent groups, two population means. Using a calculator, df is approximately 18.8462. **Do not pool the variances.**

Calculate the p-value using a Student-t distribution: p-value = 0.0054

Graph:

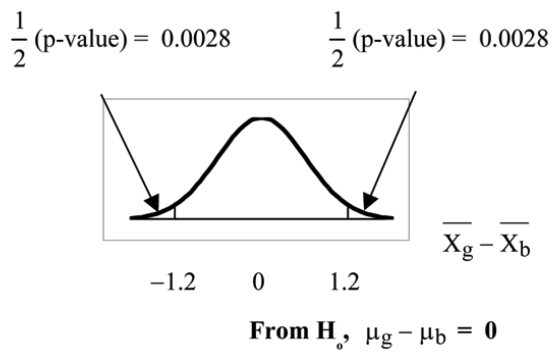

Figure 10.1

$s_g = \sqrt{0.75}$

$s_b = 1$

So, $\overline{x_g} - \overline{x_b} = 2 - 3.2 = -1.2$

Half the p-value is below -1.2 and half is above 1.2.

Make a decision: Since $\alpha >$ p-value, reject H_o.

This means you reject $\mu_g = \mu_b$. The means are different.

Conclusion: At the 5% level of significance, the sample data show there is sufficient evidence to conclude that the average number of hours that girls and boys aged 7 through 11 play sports per day is different.

NOTE: TI-83+ and TI-84: Press `STAT`. Arrow over to `TESTS` and press `4:2-SampTTest`. Arrow over to Stats and press `ENTER`. Arrow down and enter `2` for the first sample mean, $\sqrt{0.75}$ for Sx1, `9` for n1, `3.2` for the second sample mean, `1` for Sx2, and `16` for n2. Arrow down to μ1: and arrow to `does not equal` μ2. Press `ENTER`. Arrow down to Pooled: and `No`. Press `ENTER`. Arrow down to `Calculate` and press `ENTER`. The p-value is p = 0.0054, the dfs are approximately 18.8462, and the test statistic is -3.14. Do the procedure again but instead of Calculate do Draw.

Example 10.2
A study is done by a community group in two neighboring colleges to determine which one graduates students with more math classes. College A samples 11 graduates. Their average is 4 math classes with a standard deviation of 1.5 math classes. College B samples 9 graduates. Their average is 3.5 math classes with a standard deviation of 1 math class. The community group believes that a student who graduates from college A **has taken more math classes,** on the average. Test at a 1% significance level. Answer the following questions.

Problem 1 *(Solution on p. 424.)*
Is this a test of two means or two proportions?

Problem 2 *(Solution on p. 424.)*
Are the populations standard deviations known or unknown?

Problem 3 *(Solution on p. 424.)*
Which distribution do you use to perform the test?

Problem 4 *(Solution on p. 424.)*
What is the random variable?

Problem 5 *(Solution on p. 424.)*
What are the null and alternate hypothesis?

Problem 6 *(Solution on p. 424.)*
Is this test right, left, or two tailed?

Problem 7 *(Solution on p. 424.)*
What is the p-value?

Problem 8 *(Solution on p. 424.)*
Do you reject or not reject the null hypothesis?

Conclusion:
At the 1% level of significance, from the sample data, there is not sufficient evidence to conclude that a student who graduates from college A has taken more math classes, on the average, than a student who graduates from college B.

10.3 Comparing Two Independent Population Means with Known Population Standard Deviations[3]

Even though this situation is not likely (knowing the population standard deviations is not likely), the following example illustrates hypothesis testing for independent means, known population standard deviations. The distribution is Normal and is for the difference of sample means, $\overline{X_1} - \overline{X_2}$. The normal distribution has the following format:

Normal distribution

$$\overline{X_1} - \overline{X_2} \sim N\left[u_1 - u_2, \sqrt{\frac{(\sigma_1)^2}{n_1} + \frac{(\sigma_2)^2}{n_2}}\right] \quad (10.4)$$

The standard deviation is:

$$\sqrt{\frac{(\sigma_1)^2}{n_1} + \frac{(\sigma_2)^2}{n_2}} \quad (10.5)$$

The test statistic (z-score) is:

$$z = \frac{(\overline{x_1} - \overline{x_2}) - (\mu_1 - \mu_2)}{\sqrt{\frac{(\sigma_1)^2}{n_1} + \frac{(\sigma_2)^2}{n_2}}} \quad (10.6)$$

[3]This content is available online at <http://cnx.org/content/m17042/1.8/>.

Example 10.3
independent groups, population standard deviations known: The mean lasting time of 2 competing floor waxes is to be compared. **Twenty floors** are randomly assigned **to test each wax**. The following table is the result.

Wax	Sample Mean Number of Months Floor Wax Last	Population Standard Deviation
1	3	0.33
2	2.9	0.36

Table 10.2

Problem
Does the data indicate that **wax 1 is more effective than wax 2**? Test at a 5% level of significance.

Solution
This is a test of two independent groups, two population means, population standard deviations known.

Random Variable: $\overline{X_1} - \overline{X_2}$ = difference in the average number of months the competing floor waxes last.

$H_o : \mu_1 \leq \mu_2$

$H_a : \mu_1 > \mu_2$

The words **"is more effective"** says that **wax 1 lasts longer than wax 2**, on the average. "Longer" is a $'' > ''$ symbol and goes into H_a. Therefore, this is a right-tailed test.

Distribution for the test: The population standard deviations are known so the distribution is normal. Using the formula above, the distribution is:

$$\overline{X_1} - \overline{X_2} \sim N\left(0, \sqrt{\frac{0.33^2}{20} + \frac{0.36^2}{20}}\right)$$

Since $\mu_1 \leq \mu_2$ then $\mu_1 - \mu_2 \leq 0$ and the mean for the normal distribution is 0.

Calculate the p-value using the normal distribution: p-value = 0.1799

Graph:

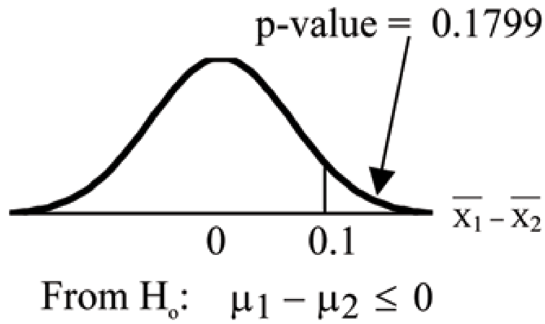

Figure 10.2

$\overline{x_1} - \overline{x_2} = 3 - 2.9 = 0.1$

Compare α and the p-value: $\alpha = 0.05$ and p-value = 0.1799. Therefore, $\alpha <$ p-value.

Make a decision: Since $\alpha <$ p-value, do not reject H_o.

Conclusion: At the 5% level of significance, from the sample data, there is not sufficient evidence to conclude that wax 1 lasts longer (wax 1 is more effective) than wax 2.

NOTE: TI-83+ and TI-84: Press `STAT`. Arrow over to `TESTS` and press `3:2-SampZTest`. Arrow over to `Stats` and press `ENTER`. Arrow down and enter `.33` for sigma1, `.36` for sigma2, `3` for the first sample mean, `20` for n1, `2.9` for the second sample mean, and `20` for n2. Arrow down to μ1: and arrow to $> \mu$2. Press `ENTER`. Arrow down to `Calculate` and press `ENTER`. The p-value is p = 0.1799 and the test statistic is 0.9157. Do the procedure again but instead of `Calculate` do `Draw`.

10.4 Comparing Two Independent Population Proportions[4]

1. The two independent samples are simple random samples that are independent.
2. The number of successes is at least five and the number of failures is at least five for each of the samples.

Comparing two proportions, like comparing two means, is common. If two estimated proportions are different, it may be due to a difference in the populations or it may be due to chance. A hypothesis test can help determine if a difference in the estimated proportions $(P'_A - P'_B)$ reflects a difference in the populations.

The difference of two proportions follows an approximate normal distribution. Generally, the null hypothesis states that the two proportions are the same. That is, $H_o : p_A = p_B$. To conduct the test, we use a pooled proportion, p_c.

[4]This content is available online at <http://cnx.org/content/m17043/1.8/>.

The pooled proportion is calculated as follows:

$$p_c = \frac{X_A + X_B}{n_A + n_B} \tag{10.7}$$

The distribution for the differences is:

$$P'_A - P'_B \sim N\left[0, \sqrt{p_c \cdot (1 - p_c) \cdot \left(\frac{1}{n_A} + \frac{1}{n_B}\right)}\right] \tag{10.8}$$

The test statistic (z-score) is:

$$z = \frac{(p'_A - p'_B) - (p_A - p_B)}{\sqrt{p_c \cdot (1 - p_c) \cdot \left(\frac{1}{n_A} + \frac{1}{n_B}\right)}} \tag{10.9}$$

Example 10.4: Two population proportions
Two types of medication for hives are being tested to determine if there is a **difference in the percentage of adult patient reactions. Twenty** out of a random **sample of 200** adults given medication A still had hives 30 minutes after taking the medication. **Twelve** out of another **random sample of 200 adults** given medication B still had hives 30 minutes after taking the medication. Test at a 1% level of significance.

10.4.1 Determining the solution

This is a test of 2 population proportions.

Problem ***(Solution on p. 424.)***

How do you know?

Let A and B be the subscripts for medication A and medication B. Then p_A and p_B are the desired population proportions.

Random Variable:
$P'_A - P'_B$ = difference in the percentages of adult patients who did not react after 30 minutes to medication A and medication B.

$H_o : p_A = p_B \qquad p_A - p_B = 0$

$H_a : p_A \neq p_B \qquad p_A - p_B \neq 0$

The words **"is a difference"** tell you the test is two-tailed.

Distribution for the test: Since this is a test of two binomial population proportions, the distribution is normal:

$p_c = \frac{X_A + X_B}{n_A + n_B} = \frac{20+12}{200+200} = 0.08 \quad 1 - p_c = 0.92$

Therefore, $P'_A - P'_B \sim N\left[0, \sqrt{(0.08) \cdot (0.92) \cdot \left(\frac{1}{200} + \frac{1}{200}\right)}\right]$

$P'_A - P'_B$ follows an approximate normal distribution.

Calculate the p-value using the normal distribution: p-value = 0.1404.

Estimated proportion for group A: $p'_A = \frac{x_A}{n_A} = \frac{20}{200} = 0.1$

Estimated proportion for group B: $p'_B = \frac{x_B}{n_B} = \frac{12}{200} = 0.06$

Graph:

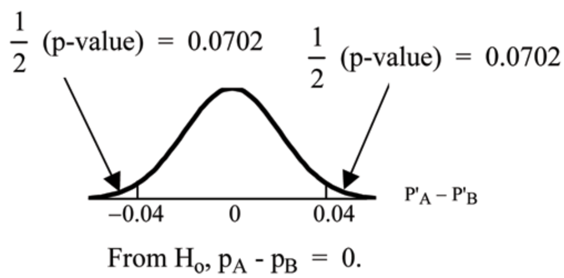

Figure 10.3

$P'_A - P'_B = 0.1 - 0.06 = 0.04$.

Half the p-value is below -0.04 and half is above 0.04.

Compare α and the p-value: $\alpha = 0.01$ and the $p\text{-}value = 0.1404$. $\alpha <$ p-value.

Make a decision: Since $\alpha < p\text{-}value$, you cannot reject H_o.

Conclusion: At a 1% level of significance, from the sample data, there is not sufficient evidence to conclude that there is a difference in the percentages of adult patients who did not react after 30 minutes to medication A and medication B.

TI-83+ and TI-84: Press `STAT`. Arrow over to `TESTS` and press `6:2-PropZTest`. Arrow down and enter `20` for $x1$, `200` for $n1$, `12` for $x2$, and `200` for $n2$. Arrow down to `p1`: and arrow to `does not equal p2`. Press `ENTER`. Arrow down to `Calculate` and press `ENTER`. The p-value is $p = 0.1404$ and the test statistic is 1.47. Do the procedure again but instead of `Calculate` do `Draw`.

10.5 Matched or Paired Samples[5]

1. Simple random sampling is used.
2. Sample sizes are often small.
3. Two measurements (samples) are drawn from the same pair of individuals or objects.
4. Differences are calculated from the matched or paired samples.
5. The differences form the sample that is used for the hypothesis test.
6. The matched pairs have differences that either come from a population that is normal or the number of differences is greater than 30 or both.

[5]This content is available online at <http://cnx.org/content/m17033/1.11/>.

In a hypothesis test for matched or paired samples, subjects are matched in pairs and differences are calculated. The differences are the data. The population mean for the differences, μ_d, is then tested using a Student-t test for a single population mean with $n-1$ degrees of freedom where n is the number of differences.

The test statistic (t-score) is:

$$t = \frac{\overline{x_d} - \mu_d}{\left(\frac{s_d}{\sqrt{n}}\right)} \tag{10.10}$$

Example 10.5: Matched or paired samples

A study was conducted to investigate the effectiveness of hypnotism in reducing pain. Results for randomly selected subjects are shown in the table. The "before" value is matched to an "after" value.

Subject:	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**
Before	6.6	6.5	9.0	10.3	11.3	8.1	6.3	11.6
After	6.8	2.5	7.4	8.5	8.1	6.1	3.4	2.0

Table 10.3

Problem

Are the sensory measurements, on average, lower after hypnotism? Test at a 5% significance level.

Solution

Corresponding "before" and "after" values form matched pairs.

After Data	**Before Data**	**Difference**
6.8	6.6	0.2
2.4	6.5	-4.1
7.4	9	-1.6
8.5	10.3	-1.8
8.1	11.3	-3.2
6.1	8.1	-2
3.4	6.3	-2.9
2	11.6	-9.6

Table 10.4

The data **for the test** are the differences: {0.2, -4.1, -1.6, -1.8, -3.2, -2, -2.9, -9.6}

The sample mean and sample standard deviation of the differences are: $\overline{x_d} = -3.13$ and $s_d = 2.91$ Verify these values.

Let μ_d be the population mean for the differences. We use the subscript d to denote "differences."

Random Variable: $\overline{X_d}$ = the average difference of the sensory measurements

$$H_o : \mu_d \geq 0 \tag{10.11}$$

There is no improvement. ($\mu_d$is the population mean of the differences.)

$$H_a : \mu_d < 0 \tag{10.12}$$

There is improvement. The score should be lower after hypnotism so the difference ought to be negative to indicate improvement.

Distribution for the test: The distribution is a student-t with $df = n - 1 = 8 - 1 = 7$. Use t_7. **(Notice that the test is for a single population mean.)**

Calculate the p-value using the Student-t distribution: *p-value* $= 0.0095$

Graph:

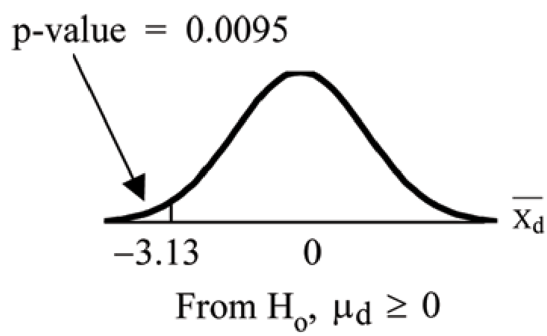

Figure 10.4

$\overline{X}_d$ is the random variable for the differences.

The sample mean and sample standard deviation of the differences are:

$\overline{x}_d = -3.13$

$\overline{s}_d = 2.91$

Compare α and the p-value: $\alpha = 0.05$ and *p-value* $= 0.0095$. $\alpha >$ *p-value*.

Make a decision: Since $\alpha >$ *p-value*, reject H_o.

This means that $\mu_d < 0$ and there is improvement.

Conclusion: At a 5% level of significance, from the sample data, there is sufficient evidence to conclude that the sensory measurements, on average, are lower after hypnotism. Hypnotism appears to be effective in reducing pain.

NOTE: For the TI-83+ and TI-84 calculators, you can either calculate the differences ahead of time (**after - before**) and put the differences into a list or you can put the **after** data into a first list and the **before** data into a second list. Then go to a third list and arrow up to the name. Enter 1st list name - 2nd list name. The calculator will do the subtraction and you will have the differences in the third list.

NOTE: TI-83+ and TI-84: Use your list of differences as the data. Press `STAT` and arrow over to `TESTS`. Press `2:T-Test`. Arrow over to `Data` and press `ENTER`. Arrow down and enter `0` for μ_0, the name of the list where you put the data, and `1` for Freq:. Arrow down to μ: and arrow over to $< \mu_0$. Press `ENTER`. Arrow down to `Calculate` and press `ENTER`. The p-value is 0.0094 and the test statistic is -3.04. Do these instructions again except arrow to `Draw` (instead of `Calculate`). Press `ENTER`.

Example 10.6

A college football coach was interested in whether the college's strength development class increased his players' maximum lift (in pounds) on the bench press exercise. He asked 4 of his players to participate in a study. The amount of weight they could each lift was recorded before they took the strength development class. After completing the class, the amount of weight they could each lift was again measured. The data are as follows:

Weight (in pounds)	Player 1	Player 2	Player 3	Player 4
Amount of weighted lifted prior to the class	205	241	338	368
Amount of weight lifted after the class	295	252	330	360

Table 10.5

The coach wants to know if the strength development class makes his players stronger, on average.

Problem *(Solution on p. 424.)*

Record the **differences** data. Calculate the differences by subtracting the amount of weight lifted prior to the class from the weight lifted after completing the class. The data for the differences are: {90, 11, -8, -8}

Using the differences data, calculate the sample mean and the sample standard deviation.

$\overline{x}_d = 21.3$ $\qquad s_d = 46.7$

Using the difference data, this becomes a test of a single __________ (fill in the blank).

Define the random variable: $\overline{X}_d$ = average difference in the maximum lift per player.

The distribution for the hypothesis test is t_3.

$H_o : \mu d \leq 0$ $\qquad H_a : \mu_d > 0$

Graph:

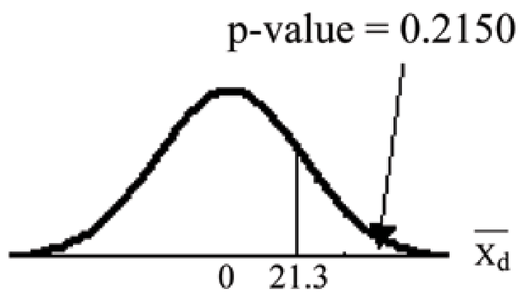

Figure 10.5

Calculate the p-value: The p-value is 0.2150

Decision: If the level of significance is 5%, the decision is to not reject the null hypothesis because $\alpha < p\text{-}value$.

What is the conclusion?

Example 10.7
Seven eighth graders at Kennedy Middle School measured how far they could push the shot-put with their dominant (writing) hand and their weaker (non-writing) hand. They thought that they could push equal distances with either hand. The following data was collected.

Distance (in feet) using	Student 1	Student 2	Student 3	Student 4	Student 5	Student 6	Student 7
Dominant Hand	30	26	34	17	19	26	20
Weaker Hand	28	14	27	18	17	26	16

Table 10.6

Problem ***(Solution on p. 424.)***
Conduct a hypothesis test to determine whether the differences in distances between the children's dominant versus weaker hands is significant.

HINT: use a t-test on the difference data.

CHECK: The test statistic is 2.18 and the p-value is 0.0716.

What is your conclusion?

10.6 Summary of Types of Hypothesis Tests[6]

Two Population Means

- Populations are independent and population standard deviations are unknown.
- Populations are independent and population standard deviations are known (not likely).

Matched or Paired Samples

- Two samples are drawn from the same set of objects.
- Samples are dependent.

Two Population Proportions

- Populations are independent.

[6]This content is available online at <http://cnx.org/content/m17044/1.5/>.

10.7 Practice 1: Hypothesis Testing for Two Proportions[7]

10.7.1 Student Learning Outcomes

- The student will explore the properties of hypothesis testing with two proportions.

10.7.2 Given

In the 2000 Census, 2.4 percent of the U.S. population reported being two or more races. However, the percent varies tremendously from state to state. (http://www.census.gov/prod/2001pubs/c2kbr01-6.pdf) Suppose that two random surveys are conducted. In the first random survey, out of 1000 North Dakotans, only 9 people reported being of two or more races. In the second random survey, out of 500 Nevadans, 17 people reported being of two or more races. Conduct a hypothesis test to determine if the population percents are the same for the two states or if the percent for Nevada is statistically higher than for North Dakota.

10.7.3 Hypothesis Testing: Two Averages

Exercise 10.7.1 *(Solution on p. 424.)*
Is this a test of averages or proportions?

Exercise 10.7.2 *(Solution on p. 424.)*
State the null and alternative hypotheses.

a. H_0 :
b. H_a :

Exercise 10.7.3 *(Solution on p. 424.)*
Is this a right-tailed, left-tailed, or two-tailed test? How do you know?

Exercise 10.7.4
What is the Random Variable of interest for this test?

Exercise 10.7.5
In words, define the Random Variable for this test.

Exercise 10.7.6 *(Solution on p. 424.)*
Which distribution (Normal or student-t) would you use for this hypothesis test?

Exercise 10.7.7
Explain why you chose the distribution you did for the above question.

Exercise 10.7.8 *(Solution on p. 424.)*
Calculate the test statistic.

Exercise 10.7.9
Sketch a graph of the situation. Label the horizontal axis. Mark the hypothesized difference and the sample difference. Shade the area corresponding to the $p-$value.

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[7]This content is available online at <http://cnx.org/content/m17027/1.9/>.

Figure 10.6

Exercise 10.7.10 ***(Solution on p. 424.)***
Find the $p-$value:

Exercise 10.7.11 ***(Solution on p. 424.)***
At a pre-conceived $\alpha = 0.05$, what is your:

- **a.** Decision:
- **b.** Reason for the decision:
- **c.** Conclusion (write out in a complete sentence):

10.7.4 Discussion Question

Exercise 10.7.12
Does it appear that the proportion of Nevadans who are two or more races is higher than the proportion of North Dakotans? Why or why not?

10.8 Practice 2: Hypothesis Testing for Two Averages[8]

10.8.1 Student Learning Outcome

- The student will explore the properties of hypothesis testing with two averages.

10.8.2 Given

The U.S. Center for Disease Control reports that the average life expectancy for whites born in 1900 was 47.6 years and for nonwhites it was 33.0 years. (http://www.cdc.gov/nchs/data/dvs/nvsr53_06t12.pdf) Suppose that you randomly survey death records for people born in 1900 in a certain county. Of the 124 whites, the average life span was 45.3 years with a standard deviation of 12.7 years. Of the 82 nonwhites, the average life span was 34.1 years with a standard deviation of 15.6 years. Conduct a hypothesis test to see if the average life spans in the county were the same for whites and nonwhites.

10.8.3 Hypothesis Testing: Two Averages

Exercise 10.8.1 *(Solution on p. 425.)*
Is this a test of averages or proportions?

Exercise 10.8.2 *(Solution on p. 425.)*
State the null and alternative hypotheses.

a. H_0 :
b. H_a :

Exercise 10.8.3 *(Solution on p. 425.)*
Is this a right-tailed, left-tailed, or two-tailed test? How do you know?

Exercise 10.8.4 *(Solution on p. 425.)*
What is the Random Variable of interest for this test?

Exercise 10.8.5 *(Solution on p. 425.)*
In words, define the Random Variable for this test.

Exercise 10.8.6
Which distribution (Normal or student-t) would you use for this hypothesis test?

Exercise 10.8.7
Explain why you chose the distribution you did for the above question.

Exercise 10.8.8 *(Solution on p. 425.)*
Calculate the test statistic.

Exercise 10.8.9
Sketch a graph of the situation. Label the horizontal axis. Mark the hypothesized difference and the sample difference. Shade the area corresponding to the $p-$value.

[8]This content is available online at <http://cnx.org/content/m17039/1.7/>.

Figure 10.7

Exercise 10.8.10 *(Solution on p. 425.)*

Find the $p-$value:

Exercise 10.8.11 *(Solution on p. 425.)*

At a pre-conceived $\alpha = 0.05$, what is your:

- **a.** Decision:
- **b.** Reason for the decision:
- **c.** Conclusion (write out in a complete sentence):

10.8.4 Discussion Question

Exercise 10.8.12

Does it appear that the averages are the same? Why or why not?

10.9 Homework[9]

For questions Exercise 10.9.1 - Exercise 10.9.10, indicate which of the following choices best identifies the hypothesis test.

A. Independent group means, population standard deviations and/or variances known
B. Independent group means, population standard deviations and/or variances unknown
C. Matched or paired samples
D. Single mean
E. 2 proportions
F. Single proportion

Exercise 10.9.1 ***(Solution on p. 425.)***
A powder diet is tested on 49 people and a liquid diet is tested on 36 different people. The population standard deviations are 2 pounds and 3 pounds, respectively. Of interest is whether the liquid diet yields a higher average weight loss than the powder diet.

Exercise 10.9.2
Two chocolate bars are taste-tested on consumers. Of interest is whether a larger percentage of consumers will prefer one bar over the other.

Exercise 10.9.3 ***(Solution on p. 425.)***
The average number of English courses taken in a two–year time period by male and female college students is believed to be about the same. An experiment is conducted and data are collected from 9 males and 16 females.

Exercise 10.9.4
A football league reported that the average number of touchdowns per game was 5. A study is done to determine if the average number of touchdowns has decreased.

Exercise 10.9.5 ***(Solution on p. 425.)***
A study is done to determine if students in the California state university system take longer to graduate than students enrolled in private universities. 100 students from both the California state university system and private universities are surveyed. From years of research, it is known that the population standard deviations are 1.5811 years and 1 year, respectively.

Exercise 10.9.6
According to a YWCA Rape Crisis Center newsletter, 75% of rape victims know their attackers. A study is done to verify this.

Exercise 10.9.7 ***(Solution on p. 425.)***
According to a recent study, U.S. companies have an average maternity-leave of six weeks.

Exercise 10.9.8
A recent drug survey showed an increase in use of drugs and alcohol among local high school students as compared to the national percent. Suppose that a survey of 100 local youths and 100 national youths is conducted to see if the percentage of drug and alcohol use is higher locally than nationally.

Exercise 10.9.9 ***(Solution on p. 425.)***
A new SAT study course is tested on 12 individuals. Pre-course and post-course scores are recorded. Of interest is the average increase in SAT scores.

Exercise 10.9.10
University of Michigan researchers reported in the *Journal of the National Cancer Institute* that quitting smoking is especially beneficial for those under age 49. In this American Cancer Society

[9]This content is available online at <http://cnx.org/content/m17023/1.10/>.

study, the risk (probability) of dying of lung cancer was about the same as for those who had never smoked.

10.9.1 For each problem below, fill in a hypothesis test solution sheet. The solution sheet is in the Appendix and can be copied. For the online version of the book, it is suggested that you copy the .doc or .pdf files.

NOTE: If you are using a student-t distribution for a homework problem below, including for paired data, you may assume that the underlying population is normally distributed. (In general, you must first prove that assumption, though.)

Exercise 10.9.11 ***(Solution on p. 425.)***

A powder diet is tested on 49 people and a liquid diet is tested on 36 different people. Of interest is whether the liquid diet yields a higher average weight loss than the powder diet. The powder diet group had an average weight loss of 42 pounds with a standard deviation of 12 pounds. The liquid diet group had an average weight loss of 45 pounds with a standard deviation of 14 pounds.

Exercise 10.9.12

The average number of English courses taken in a two–year time period by male and female college students is believed to be about the same. An experiment is conducted and data are collected from 29 males and 16 females. The males took an average of 3 English courses with a standard deviation of 0.8. The females took an average of 4 English courses with a standard deviation of 1.0. Are the averages statistically the same?

Exercise 10.9.13 ***(Solution on p. 425.)***

A study is done to determine if students in the California state university system take longer to graduate than students enrolled in private universities. 100 students from both the California state university system and private universities are surveyed. Suppose that from years of research, it is known that the population standard deviations are 1.5811 years and 1 year, respectively. The following data are collected. The California state university system students took on average 4.5 years with a standard deviation of 0.8. The private university students took on average 4.1 years with a standard deviation of 0.3.

Exercise 10.9.14

A new SAT study course is tested on 12 individuals. Pre-course and post-course scores are recorded. Of interest is the average increase in SAT scores. The following data is collected:

Pre-course score	Post-course score
1200	1300
960	920
1010	1100
840	880
1100	1070
1250	1320
860	860
1330	1370
790	770
990	1040
1110	1200
740	850

Table 10.7

Exercise 10.9.15 ***(Solution on p. 425.)***

A recent drug survey showed an increase in use of drugs and alcohol among local high school seniors as compared to the national percent. Suppose that a survey of 100 local seniors and 100 national seniors is conducted to see if the percentage of drug and alcohol use is higher locally than nationally. Locally, 65 seniors reported using drugs or alcohol within the past month, while 60 national seniors reported using them.

Exercise 10.9.16

A student at a four-year college claims that average enrollment at four–year colleges is higher than at two–year colleges in the United States. Two surveys are conducted. Of the 35 two–year colleges surveyed, the average enrollment was 5068 with a standard deviation of 4777. Of the 35 four-year colleges surveyed, the average enrollment was 5466 with a standard deviation of 8191. (Source: *Microsoft Bookshelf*)

Exercise 10.9.17 ***(Solution on p. 426.)***

A study was conducted by the U.S. Army to see if applying antiperspirant to soldiers' feet for a few days before a major hike would help cut down on the number of blisters soldiers had on their feet. In the experiment, for three nights before they went on a 13-mile hike, a group of 328 West Point cadets put an alcohol-based antiperspirant on their feet. A "control group" of 339 soldiers put on a similar, but inactive, preparation on their feet. On the day of the hike, the temperature reached 83 ° F. At the end of the hike, 21% of the soldiers who had used the antiperspirant and 48% of the control group had developed foot blisters. Conduct a hypothesis test to see if the percent of soldiers using the antiperspirant was significantly lower than the control group. (Source: U.S. Army study reported in *Journal of the American Academy of Dermatologists*)

Exercise 10.9.18

We are interested in whether the percents of female suicide victims for ages 15 to 24 are the same for the white and the black races in the United States. We randomly pick one year, 1992, to compare the races. The number of suicides estimated in the United States in 1992 for white females is 4930. 580 were aged 15 to 24. The estimate for black females is 330. 40 were aged 15 to 24. We will let female suicide victims be our population. (Source*: the National Center for Health Statistics, U.S. Dept. of Health and Human Services*)

Exercise 10.9.19 *(Solution on p. 426.)*

At Rachel's 11th birthday party, 8 girls were timed to see how long (in seconds) they could hold their breath in a relaxed position. After a two-minute rest, they timed themselves while jumping. The girls thought that the jumping would not affect their times, on average. Test their hypothesis.

Relaxed time (seconds)	Jumping time (seconds)
26	21
47	40
30	28
22	21
23	25
45	43
37	35
29	32

Table 10.8

Exercise 10.9.20

Elizabeth Mjelde, an art history professor, was interested in whether the value from the Golden Ratio formula, $\left(\frac{\textit{larger+smaller dimension}}{\textit{larger dimension}}\right)$was the same in the Whitney Exhibit for works from 1900 – 1919 as for works from 1920 – 1942. 37 early works were sampled. They averaged 1.74 with a standard deviation of 0.11. 65 of the later works were sampled. They averaged 1.746 with a standard deviation of 0.1064. Do you think that there is a significant difference in the Golden Ratio calculation? (Source: *data from Whitney Exhibit on loan to San Jose Museum of Art*)

Exercise 10.9.21 *(Solution on p. 426.)*

One of the questions in a study of marital satisfaction of dual–career couples was to rate the statement, "I'm pleased with the way we divide the responsibilities for childcare." The ratings went from 1 (strongly agree) to 5 (strongly disagree). Below are ten of the paired responses for husbands and wives. Conduct a hypothesis test to see if the average difference in the husband's versus the wife's satisfaction level is negative (meaning that, within the partnership, the husband is happier than the wife).

Wife's score	2	2	3	3	4	2	1	1	2	4
Husband's score	2	2	1	3	2	1	1	1	2	4

Table 10.9

Exercise 10.9.22

Ten individuals went on a low–fat diet for 12 weeks to lower their cholesterol. Evaluate the data below. Do you think that their cholesterol levels were significantly lowered?

Starting cholesterol level	Ending cholesterol level
140	140
220	230
110	120
240	220
200	190
180	150
190	200
360	300
280	300
260	240

Table 10.10

Exercise 10.9.23 ***(Solution on p. 426.)***

Average entry level salaries for college graduates with mechanical engineering degrees and electrical engineering degrees are believed to be approximately the same. (Source: *http://www.graduatingengineer.com*[10]). A recruiting office thinks that the average mechanical engineering salary is actually lower than the average electrical engineering salary. The recruiting office randomly surveys 50 entry level mechanical engineers and 60 entry level electrical engineers. Their average salaries were \$46,100 and \$46,700, respectively. Their standard deviations were \$3450 and \$4210, respectively. Conduct a hypothesis test to determine if you agree that the average entry level mechanical engineering salary is lower than the average entry level electrical engineering salary.

Exercise 10.9.24

A recent year was randomly picked from 1985 to the present. In that year, there were 2051 Hispanic students at Cabrillo College out of a total of 12,328 students. At Lake Tahoe College, there were 321 Hispanic students out of a total of 2441 students. In general, do you think that the percent of Hispanic students at the two colleges is basically the same or different? (Source: *Chancellor's Office, California Community Colleges, November 1994*)

Exercise 10.9.25 ***(Solution on p. 426.)***

Eight runners were convinced that the average difference in their individual times for running one mile versus race walking one mile was at most 2 minutes. Below are their times. Do you agree that the average difference is at most 2 minutes?

[10]http://www.graduatingengineer.com/

Running time (minutes)	Race walking time (minutes)
5.1	7.3
5.6	9.2
6.2	10.4
4.8	6.9
7.1	8.9
4.2	9.5
6.1	9.4
4.4	7.9

Table 10.11

Exercise 10.9.26
Marketing companies have collected data implying that teenage girls use more ring tones on their cellular phones than teenage boys do. In one particular study of 40 randomly chosen teenage girls and boys (20 of each) with cellular phones, the average number of ring tones for the girls was 3.2 with a standard deviation of 1.5. The average for the boys was 1.7 with a standard deviation of 0.8. Conduct a hypothesis test to determine if the averages are approximately the same or if the girls' average is higher than the boys' average.

Exercise 10.9.27 ***(Solution on p. 426.)***
While her husband spent 2½ hours picking out new speakers, a statistician decided to determine whether the percent of men who enjoy shopping for electronic equipment is higher than the percent of women who enjoy shopping for electronic equipment. The population was Saturday afternoon shoppers. Out of 67 men, 24 said they enjoyed the activity. 8 of the 24 women surveyed claimed to enjoy the activity. Interpret the results of the survey.

Exercise 10.9.28
We are interested in whether children's educational computer software costs less, on average, than children's entertainment software. 36 educational software titles were randomly picked from a catalog. The average cost was $31.14 with a standard deviation of $4.69. 35 entertainment software titles were randomly picked from the same catalog. The average cost was $33.86 with a standard deviation of $10.87. Decide whether children's educational software costs less, on average, than children's entertainment software. (Source: *Educational Resources*, December catalog)

Exercise 10.9.29 ***(Solution on p. 426.)***
Parents of teenage boys often complain that auto insurance costs more, on average, for teenage boys than for teenage girls. A group of concerned parents examines a random sample of insurance bills. The average annual cost for 36 teenage boys was $679. For 23 teenage girls, it was $559. From past years, it is known that the population standard deviation for each group is $180. Determine whether or not you believe that the average cost for auto insurance for teenage boys is greater than that for teenage girls.

Exercise 10.9.30
A group of transfer bound students wondered if they will spend the same average amount on texts and supplies each year at their four-year university as they have at their community college. They conducted a random survey of 54 students at their community college and 66 students at their local four-year university. The sample means were $947 and $1011, respectively. The population standard deviations are known to be $254 and $87, respectively. Conduct a hypothesis test to determine if the averages are statistically the same.

Exercise 10.9.31 *(Solution on p. 426.)*
Joan Nguyen recently claimed that the proportion of college–age males with at least one pierced ear is as high as the proportion of college–age females. She conducted a survey in her classes. Out of 107 males, 20 had at least one pierced ear. Out of 92 females, 47 had at least one pierced ear. Do you believe that the proportion of males has reached the proportion of females?

Exercise 10.9.32
Some manufacturers claim that non-hybrid sedan cars have a lower average miles per gallon (mpg) than hybrid ones. Suppose that consumers test 21 hybrid sedans and get an average 31 mpg with a standard deviation of 7 mpg. Thirty-one non-hybrid sedans average 22 mpg with a standard deviation of 4 mpg. Suppose that the population standard deviations are known to be 6 and 3, respectively. Conduct a hypothesis test to the manufacturers claim.

Questions Exercise 10.9.33 – Exercise 10.9.37 refer to the Terri Vogel's data set (see Table of Contents).

Exercise 10.9.33 *(Solution on p. 426.)*
Using the data from Lap 1 only, conduct a hypothesis test to determine if the average time for completing a lap in races is the same as it is in practices.

Exercise 10.9.34
Repeat the test in Exercise 10.9.33, but use Lap 5 data this time.

Exercise 10.9.35 *(Solution on p. 427.)*
Repeat the test in Exercise 10.9.33, but this time combine the data from Laps 1 and 5.

Exercise 10.9.36
In 2 – 3 complete sentences, explain in detail how you might use Terri Vogel's data to answer the following question. "Does Terri Vogel drive faster in races than she does in practices?"

Exercise 10.9.37 *(Solution on p. 427.)*
Is the proportion of race laps Terri completes slower than 130 seconds less than the proportion of practice laps she completes slower than 135 seconds?

Exercise 10.9.38
"To Breakfast or Not to Breakfast?" by Richard Ayore

In the American society, birthdays are one of those days that everyone looks forward to. People of different ages and peer groups gather to mark the 18th, 20th, . . . birthdays. During this time, one looks back to see what he or she had achieved for the past year, and also focuses ahead for more to come.

If, by any chance, I am invited to one of these parties, my experience is always different. Instead of dancing around with my friends while the music is booming, I get carried away by memories of my family back home in Kenya. I remember the good times I had with my brothers and sister while we did our daily routine.

Every morning, I remember we went to the shamba (garden) to weed our crops. I remember one day arguing with my brother as to why he always remained behind just to join us an hour later. In his defense, he said that he preferred waiting for breakfast before he came to weed. He said, "This is why I always work more hours than you guys!"

And so, to prove his wrong or right, we decided to give it a try. One day we went to work as usual without breakfast, and recorded the time we could work before getting tired and stopping. On the next day, we all ate breakfast before going to work. We recorded how long we worked again before getting tired and stopping. Of interest was our average increase in work time. Though not sure, my brother insisted that it is more than two hours. Using the data below, solve our problem.

Work hours with breakfast	Work hours without breakfast
8	6
7	5
9	5
5	4
9	7
8	7
10	7
7	5
6	6
9	5

Table 10.12

10.9.2 Try these multiple choice questions.

For questions Exercise 10.9.39 – Exercise 10.9.40, use the following information.

A new AIDS prevention drugs was tried on a group of 224 HIV positive patients. Forty-five (45) patients developed AIDS after four years. In a control group of 224 HIV positive patients, 68 developed AIDS after four years. We want to test whether the method of treatment reduces the proportion of patients that develop AIDS after four years or if the proportions of the treated group and the untreated group stay the same.

Let the subscript t= treated patient and ut= untreated patient.

Exercise 10.9.39 ***(Solution on p. 427.)***

The appropriate hypotheses are:

A. $H_o : p_t < p_{ut}$ and $H_a : p_t \geq p_{ut}$
B. $H_o : p_t \leq p_{ut}$ and $H_a : p_t > p_{ut}$
C. $H_o : p_t = p_{ut}$ and $H_a : p_t \neq p_{ut}$
D. $H_o : p_t = p_{ut}$ and $H_a : p_t < p_{ut}$

Exercise 10.9.40 ***(Solution on p. 427.)***

If the p -value is 0.0062 what is the conclusion (use $\alpha = 5$)?

A. The method has no effect.
B. The method reduces the proportion of HIV positive patients that develop AIDS after four years.
C. The method increases the proportion of HIV positive patients that develop AIDS after four years.
D. The test does not determine whether the method helps or does not help.

Exercise 10.9.41 ***(Solution on p. 427.)***

Lesley E. Tan investigated the relationship between left-handedness and right-handedness and motor competence in preschool children. Random samples of 41 left-handers and 41 right-handers were given several tests of motor skills to determine if there is evidence of a difference between the children based on this experiment. The experiment produced the means and standard deviations shown below. Determine the appropriate test and best distribution to use for that test.

	Left-handed	Right-handed
Sample size	41	41
Sample mean	97.5	98.1
Sample standard deviation	17.5	19.2

Table 10.13

A. Two independent means, normal distribution
B. Two independent means, student-t distribution
C. Matched or paired samples, student-t distribution
D. Two population proportions, normal distribution

For questions Exercise 10.9.42 – Exercise 10.9.43, use the following information.

An experiment is conducted to show that blood pressure can be consciously reduced in people trained in a "biofeedback exercise program." Six (6) subjects were randomly selected and the blood pressure measurements were recorded before and after the training. The difference between blood pressures was calculated $(after - before)$ producing the following results: $\overline{x}_d = -10.2$ $s_d = 8.4$. Using the data, test the hypothesis that the blood pressure has decreased after the training,

Exercise 10.9.42 *(Solution on p. 427.)*

The distribution for the test is

A. t_5
B. t_6
C. $N(-10.2, 8.4)$
D. $N\left(-10.2, \frac{8.4}{\sqrt{6}}\right)$

Exercise 10.9.43 *(Solution on p. 427.)*

If $\alpha = 0.05$, the p-value and the conclusion are

A. 0.0014; the blood pressure decreased after the training
B. 0.0014; the blood pressure increased after the training
C. 0.0155; the blood pressure decreased after the training
D. 0.0155; the blood pressure increased after the training

For questions Exercise 10.9.44– Exercise 10.9.45, use the following information.

The Eastern and Western Major League Soccer conferences have a new Reserve Division that allows new players to develop their skills. As of May 25, 2005, the Reserve Division teams scored the following number of goals for 2005.

Western	Eastern
Los Angeles 9	D.C. United 9
FC Dallas 3	Chicago 8
Chivas USA 4	Columbus 7
Real Salt Lake 3	New England 6
Colorado 4	MetroStars 5
San Jose 4	Kansas City 3

Table 10.14

Conduct a hypothesis test to determine if the Western Reserve Division teams score, on average, fewer goals than the Eastern Reserve Division teams. Subscripts: **1** Western Reserve Division (**W**); **2** Eastern Reserve Division (**E**)

Exercise 10.9.44 *(Solution on p. 427.)*

The **exact** distribution for the hypothesis test is:

A. The normal distribution.
B. The student-t distribution.
C. The uniform distribution.
D. The exponential distribution.

Exercise 10.9.45 *(Solution on p. 427.)*

If the level of significance is 0.05, the conclusion is:

A. The **W** Division teams score, on average, fewer goals than the **E** teams.
B. The **W** Division teams score, on average, more goals than the **E** teams.
C. The **W** teams score, on average, about the same number of goals as the **E** teams score.
D. Unable to determine.

Questions Exercise 10.9.46 – Exercise 10.9.48 refer to the following.

A researcher is interested in determining if a certain drug vaccine prevents West Nile disease. The vaccine with the drug is administered to 36 people and another 36 people are given a vaccine that does not contain the drug. Of the group that gets the vaccine with the drug, one (1) gets West Nile disease. Of the group that gets the vaccine without the drug, three (3) get West Nile disease. Conduct a hypothesis test to determine if the proportion of people that get the vaccine without the drug and get West Nile disease is more than the proportion of people that get the vaccine with the drug and get West Nile disease.

- "Drug" subscript: group who get the vaccine with the drug.
- "No Drug" subscript: group who get the vaccine without the drug

Exercise 10.9.46 *(Solution on p. 427.)*

This is a test of:

A. a test of two proportions
B. a test of two independent means
C. a test of a single mean
D. a test of matched pairs.

Exercise 10.9.47 *(Solution on p. 427.)*

An appropriate null hypothesis is:

A. $p_{No\ Drug} \leq p_{Drug}$
B. $p_{No\ Drug} \geq p_{Drug}$
C. $\mu_{No\ Drug} \leq \mu_{Drug}$
D. $p_{No\ Drug} > p_{Drug}$

Exercise 10.9.48 ***(Solution on p. 427.)***

The p-value is 0.1517. At a 1% level of significance, the appropriate conclusion is

A. the proportion of people that get the vaccine without the drug and get West Nile disease is less than the proportion of people that get the vaccine with the drug and get West Nile disease.
B. the proportion of people that get the vaccine without the drug and get West Nile disease is more than the proportion of people that get the vaccine with the drug and get West Nile disease.
C. the proportion of people that get the vaccine without the drug and get West Nile disease is more than or equal to the proportion of people that get the vaccine with the drug and get West Nile disease.
D. the proportion of people that get the vaccine without the drug and get West Nile disease is no more than the proportion of people that get the vaccine with the drug and get West Nile disease.

Questions Exercise 10.9.49 and Exercise 10.9.50 refer to the following:

A golf instructor is interested in determining if her new technique for improving players' golf scores is effective. She takes four (4) new students. She records their 18-holes scores before learning the technique and then after having taken her class. She conducts a hypothesis test. The data are as follows.

	Player 1	Player 2	Player 3	Player 4
Average score before class	83	78	93	87
Average score after class	80	80	86	86

Table 10.15

Exercise 10.9.49 ***(Solution on p. 427.)***

This is a test of:

A. a test of two independent means
B. a test of two proportions
C. a test of a single proportion
D. a test of matched pairs.

Exercise 10.9.50 ***(Solution on p. 427.)***

The correct decision is:

A. Reject H_o
B. Do not reject H_o
C. The test is inconclusive

Questions Exercise 10.9.51 and Exercise 10.9.52 refer to the following:

Suppose a statistics instructor believes that there is no significant difference between the average class scores of her two classes on Exam 2. The average and standard deviation for her 8:30 class of 35 students were 75.86 and 16.91. The average and standard deviation for her 11:30 class of 37 students were 75.41 and 19.73. "8:30" subscript refers to the 8:30 class. "11:30" subscript refers to the 11:30 class.

Exercise 10.9.51 *(Solution on p. 427.)*

An appropriate alternate hypothesis for the hypothesis test is:

A. $\mu_{8:30} > \mu_{11:30}$
B. $\mu_{8:30} < \mu_{11:30}$
C. $\mu_{8:30} = \mu_{11:30}$
D. $\mu_{8:30} \neq \mu_{11:30}$

Exercise 10.9.52 *(Solution on p. 427.)*

A concluding statement is:

A. The 11:30 class average is better than the 8:30 class average.
B. The 8:30 class average is better than the 11:30 class average.
C. There is no significant difference between the averages of the two classes.
D. There is a significant difference between the averages of the two classes.

10.10 Review[11]

The next three questions refer to the following information:

In a survey at Kirkwood Ski Resort the following information was recorded:

Sport Participation by Age

	0 – 10	11 - 20	21 - 40	40+
Ski	10	12	30	8
Snowboard	6	17	12	5

Table 10.16

Suppose that one person from of the above was randomly selected.

Exercise 10.10.1 *(Solution on p. 427.)*

Find the probability that the person was a skier or was age 11 – 20.

Exercise 10.10.2 *(Solution on p. 427.)*

Find the probability that the person was a snowboarder given he/she was age 21 – 40.

Exercise 10.10.3 *(Solution on p. 427.)*

Explain which of the following are true and which are false.

a. Sport and Age are independent events.
b. Ski and age 11 – 20 are mutually exclusive events.
c. $P\left(\textit{Ski and age } 21 - 40\right) < P\left(\textit{Ski} \mid \textit{age } 21 - 40\right)$
d. $P\left(\textit{Snowboard or age } 0 - 10\right) < P\left(\textit{Snowboard} \mid \textit{age } 0 - 10\right)$

Exercise 10.10.4 *(Solution on p. 428.)*

The average length of time a person with a broken leg wears a cast is approximately 6 weeks. The standard deviation is about 3 weeks. Thirty people who had recently healed from broken legs were interviewed. State the distribution that most accurately reflects total time to heal for the thirty people.

Exercise 10.10.5 *(Solution on p. 428.)*

The distribution for X is Uniform. What can we say for certain about the distribution for $\overline{X}$ when $n = 1$?

A. The distribution for $\overline{X}$ is still Uniform with the same mean and standard dev. as the distribution for X.
B. The distribution for $\overline{X}$is Normal with the different mean and a different standard deviation as the distribution for X.
C. The distribution for $\overline{X}$ is Normal with the same mean but a larger standard deviation than the distribution for X.
D. The distribution for $\overline{X}$ is Normal with the same mean but a smaller standard deviation than the distribution for X.

Exercise 10.10.6 *(Solution on p. 428.)*

The distribution for X is uniform. What can we say for certain about the distribution for $\sum X$ when $n = 50$?

[11] This content is available online at <http://cnx.org/content/m17021/1.8/>.

A. The distribution for $\sum X$is still uniform with the same mean and standard deviation as the distribution for X.
B. The distribution for $\sum X$ is Normal with the same mean but a larger standard deviation as the distribution for X.
C. The distribution for $\sum X$ is Normal with a larger mean and a larger standard deviation than the distribution for X.
D. The distribution for $\sum X$ is Normal with the same mean but a smaller standard deviation than the distribution for X.

The next three questions refer to the following information:

A group of students measured the lengths of all the carrots in a five-pound bag of baby carrots. They calculated the average length of baby carrots to be 2.0 inches with a standard deviation of 0.25 inches. Suppose we randomly survey 16 five-pound bags of baby carrots.

Exercise 10.10.7 ***(Solution on p. 428.)***

State the approximate distribution for $\overline{X}$, the distribution for the average lengths of baby carrots in 16 five-pound bags. $\overline{X}$~

Exercise 10.10.8

Explain why we cannot find the probability that one individual randomly chosen carrot is greater than 2.25 inches.

Exercise 10.10.9 ***(Solution on p. 428.)***

Find the probability that $\overline{X}$ is between 2 and 2.25 inches.

The next three questions refer to the following information:

At the beginning of the term, the amount of time a student waits in line at the campus store is normally distributed with a mean of 5 minutes and a standard deviation of 2 minutes.

Exercise 10.10.10 ***(Solution on p. 428.)***

Find the 90th percentile of waiting time in minutes.

Exercise 10.10.11 ***(Solution on p. 428.)***

Find the median waiting time for one student.

Exercise 10.10.12 ***(Solution on p. 428.)***

Find the probability that the average waiting time for 40 students is at least 4.5 minutes.

10.11 Lab: Hypothesis Testing for Two Means and Two Proportions[12]

Class Time:

Names:

10.11.1 Student Learning Outcomes:

- The student will select the appropriate distributions to use in each case.
- The student will conduct hypothesis tests and interpret the results.

10.11.2 Supplies:

- The business section from two consecutive days' newspapers
- 3 small packages of M&Ms®
- 5 small packages of Reeses Pieces®

10.11.3 Increasing Stocks Survey

Look at yesterday's newspaper business section. Conduct a hypothesis test to determine if the proportion of New York Stock Exchange (NYSE) stocks that increased is greater than the proportion of NASDAQ stocks that increased. As randomly as possible, choose 40 NYSE stocks and 32 NASDAQ stocks and complete the following statements.

1. H_o
2. H_a
3. In words, define the Random Variable. ____________=
4. The distribution to use for the test is:
5. Calculate the test statistic using your data.
6. Draw a graph and label it appropriately. Shade the actual level of significance.
   a. Graph:

[12]This content is available online at <http://cnx.org/content/m17022/1.10/>.

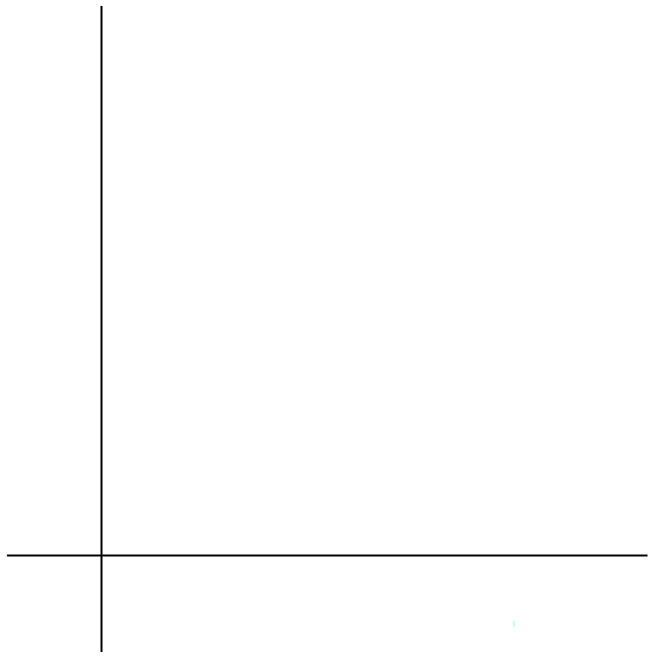

Figure 10.8

b. Calculate the p-value:

7. Do you reject or not reject the null hypothesis? Why?
8. Write a clear conclusion using a complete sentence.

10.11.4 Decreasing Stocks Survey

Randomly pick 8 stocks from the newspaper. Using two consecutive days' business sections, test whether the stocks went down, on average, for the second day.

1. H_o
2. H_a
3. In words, define the Random Variable. ____________=
4. The distribution to use for the test is:
5. Calculate the test statistic using your data.
6. Draw a graph and label it appropriately. Shade the actual level of significance.

   a. Graph:

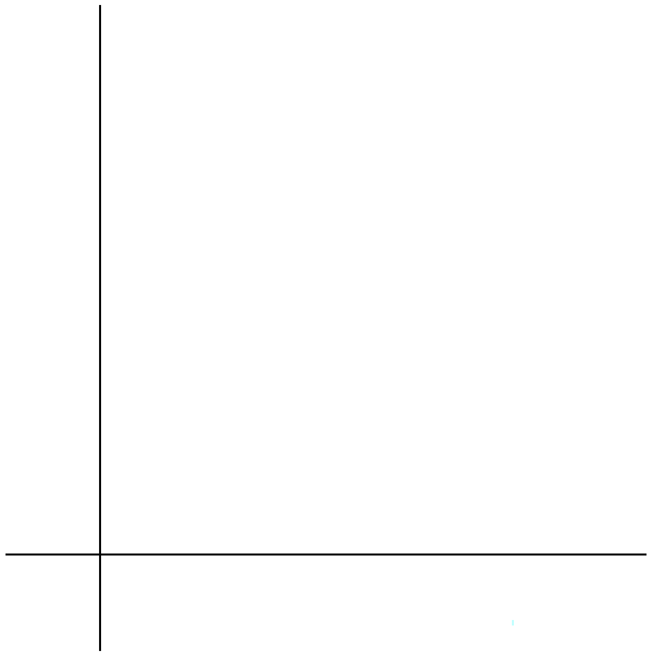

Figure 10.9

b. Calculate the p-value:

7. Do you reject or not reject the null hypothesis? Why?
8. Write a clear conclusion using a complete sentence.

10.11.5 Candy Survey

Buy three small packages of M&Ms and 5 small packages of Reeses Pieces (same net weight as the M&Ms). Test whether or not the average number of candy pieces per package is the same for the two brands.

1. H_o:
2. H_a:
3. In words, define the random variable. __________=
4. What distribution should be used for this test?
5. Calculate the test statistic using your data.
6. Draw a graph and label it appropriately. Shade the actual level of significance.

   a. Graph:

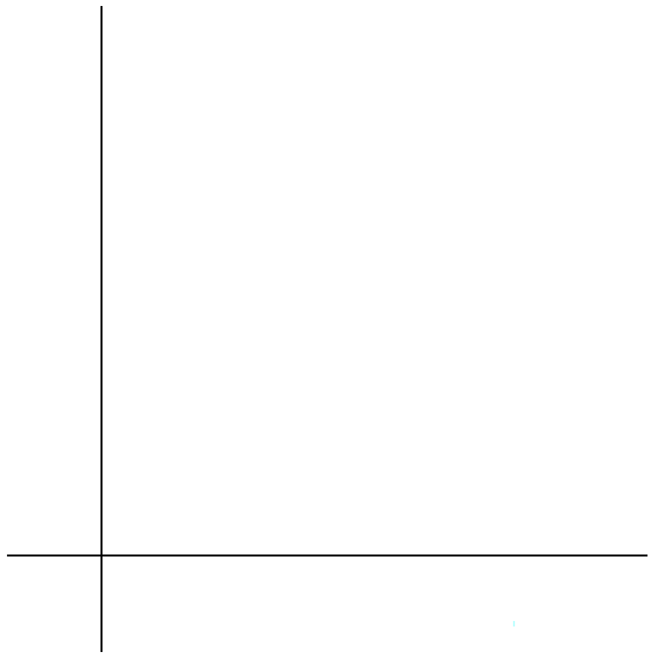

Figure 10.10

b. Calculate the p-value:

7. Do you reject or not reject the null hypothesis? Why?
8. Write a clear conclusion using a complete sentence.

10.11.6 Shoe Survey

Test whether women have, on average, more pairs of shoes than men. Include all forms of sneakers, shoes, sandals, and boots. Use your class as the sample.

1. H_o
2. H_a
3. In words, define the Random Variable. ____________=
4. The distribution to use for the test is:
5. Calculate the test statistic using your data.
6. Draw a graph and label it appropriately. Shade the actual level of significance.

a. Graph:

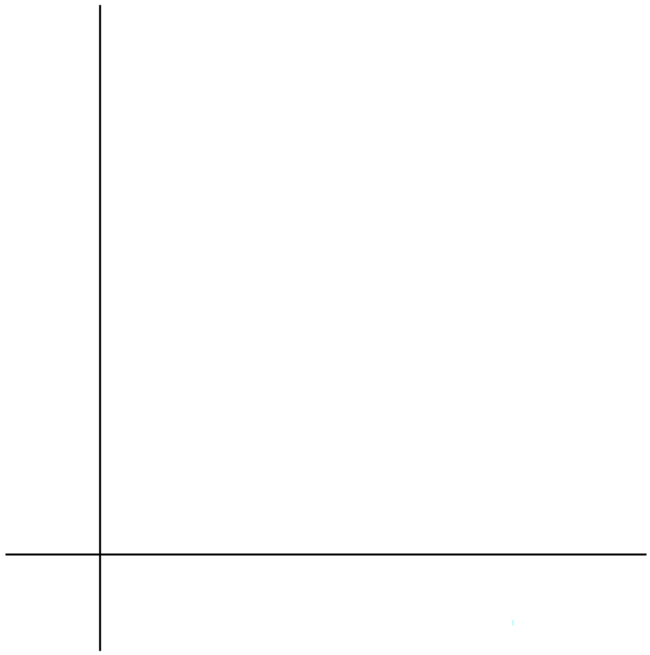

Figure 10.11

b. Calculate the p-value:

7. Do you reject or not reject the null hypothesis? Why?
8. Write a clear conclusion using a complete sentence.

Solutions to Exercises in Chapter 10

Solution to Example 10.2, Problem 1 (p. 391)
two means
Solution to Example 10.2, Problem 2 (p. 391)
unknown
Solution to Example 10.2, Problem 3 (p. 391)
Student-t
Solution to Example 10.2, Problem 4 (p. 391)
$\overline{X_A} - \overline{X_B}$
Solution to Example 10.2, Problem 5 (p. 391)

- $H_o : \mu_A \leq \mu_B$
- $H_a : \mu_A > \mu_B$

Solution to Example 10.2, Problem 6 (p. 391)
right
Solution to Example 10.2, Problem 7 (p. 391)
0.1928
Solution to Example 10.2, Problem 8 (p. 391)
Do not reject.
Solution to Example 10.4 (p. 394)
The problem asks for a difference in percentages.
Solution to Example 10.6 (p. 398)
means; At a 5% level of significance, from the sample data, there is not sufficient evidence to conclude that the strength development class helped to make the players stronger, on average.
Solution to Example 10.7 (p. 399)
H_0: μ_d equals 0; H_a: μ_d does not equal 0; Do not reject the null; At a 5% significance level, from the sample data, there is not sufficient evidence to conclude that the differences in distances between the children's dominant versus weaker hands is significant (there is not sufficient evidence to show that the children could push the shot-put further with their dominant hand). Alpha and the p-value are close so the test is not strong.

Solutions to Practice 1: Hypothesis Testing for Two Proportions

Solution to Exercise 10.7.1 (p. 401)
Proportions
Solution to Exercise 10.7.2 (p. 401)

a. H_0:$_{PN}$=$_{PND}$

a. H_a:$_{PN}$> $_{PND}$

Solution to Exercise 10.7.3 (p. 401)
right-tailed
Solution to Exercise 10.7.6 (p. 401)
Normal
Solution to Exercise 10.7.8 (p. 401)
3.50
Solution to Exercise 10.7.10 (p. 402)
0.0002
Solution to Exercise 10.7.11 (p. 402)

a. Reject the null hypothesis

Solutions to Practice 2: Hypothesis Testing for Two Averages

Solution to Exercise 10.8.1 (p. 403)
Averages
Solution to Exercise 10.8.2 (p. 403)

- **a.** $H_0 : \mu_W = \mu_{NW}$
- **b.** $H_a : \mu_W \neq \mu_{NW}$

Solution to Exercise 10.8.3 (p. 403)
two-tailed
Solution to Exercise 10.8.4 (p. 403)
$\overline{X}_W - \overline{X}_{NW}$
Solution to Exercise 10.8.5 (p. 403)
student-t
Solution to Exercise 10.8.8 (p. 403)
5.42
Solution to Exercise 10.8.10 (p. 404)
0.0000
Solution to Exercise 10.8.11 (p. 404)

- **a.** Reject the null hypothesis

Solutions to Homework

Solution to Exercise 10.9.1 (p. 405)
A
Solution to Exercise 10.9.3 (p. 405)
B
Solution to Exercise 10.9.5 (p. 405)
A
Solution to Exercise 10.9.7 (p. 405)
D
Solution to Exercise 10.9.9 (p. 405)
C
Solution to Exercise 10.9.11 (p. 406)

- **d.** $t_{68.44}$
- **e.** -1.04
- **f.** 0.1519
- **h.** Dec: do not reject null

Solution to Exercise 10.9.13 (p. 406)
Standard Normal

- **e.** $z = 2.14$
- **f.** 0.0163
- **h.** Decision: Reject null when $\alpha = 0.05$; Do not reject null when $\alpha = 0.01$

Solution to Exercise 10.9.15 (p. 407)

- **e.** 0.73
- **f.** 0.2326
- **h.** Decision: Do not reject null

Solution to Exercise 10.9.17 (p. 407)

e. -7.33
f. 0
h. Decision: Reject null

Solution to Exercise 10.9.19 (p. 408)

d. t_7
e. -1.51
f. 0.1755
h. Decision: Do not reject null

Solution to Exercise 10.9.21 (p. 408)

d. t_9
e. $t = -1.86$
f. 0.0479
h. Decision: Reject null, but run another test

Solution to Exercise 10.9.23 (p. 409)

d. t_{108}
e. $t = -0.82$
f. 0.2066
h. Decision: Do not reject null

Solution to Exercise 10.9.25 (p. 409)

d. t_7
e. $t = 2.9850$
f. 0.0103
h. Decision: Reject null; The average difference is more than 2 minutes.

Solution to Exercise 10.9.27 (p. 410)

e. 0.22
f. 0.4133
h. Decision: Do not reject null

Solution to Exercise 10.9.29 (p. 410)

e. $z = 2.50$
f. 0.0063
h. Decision: Reject null

Solution to Exercise 10.9.31 (p. 411)

e. -4.82
f. 0
h. Decision: Reject null

Solution to Exercise 10.9.33 (p. 411)

d. $t_{20.32}$
e. -4.70
f. 0.0001
h. Decision: Reject null

Solution to Exercise 10.9.35 (p. 411)

d. $t_{40.94}$
e. -5.08
f. 0
h. Decision: Reject null

Solution to Exercise 10.9.37 (p. 411)

e. -0.95
f. 0.1705
h. Decision: Do not reject null

Solution to Exercise 10.9.39 (p. 412)
D
Solution to Exercise 10.9.40 (p. 412)
B
Solution to Exercise 10.9.41 (p. 412)
B
Solution to Exercise 10.9.42 (p. 413)
A
Solution to Exercise 10.9.43 (p. 413)
C
Solution to Exercise 10.9.44 (p. 414)
B
Solution to Exercise 10.9.45 (p. 414)
C
Solution to Exercise 10.9.46 (p. 414)
A
Solution to Exercise 10.9.47 (p. 414)
A
Solution to Exercise 10.9.48 (p. 415)
D
Solution to Exercise 10.9.49 (p. 415)
D
Solution to Exercise 10.9.50 (p. 415)
B
Solution to Exercise 10.9.51 (p. 416)
D
Solution to Exercise 10.9.52 (p. 416)
C

Solutions to Review

Solution to Exercise 10.10.1 (p. 417)
$\frac{77}{100}$
Solution to Exercise 10.10.2 (p. 417)
$\frac{12}{42}$
Solution to Exercise 10.10.3 (p. 417)

a. False
b. False
c. True

d. False

Solution to Exercise 10.10.4 (p. 417)
$N(180, 16.43)$
Solution to Exercise 10.10.5 (p. 417)
A
Solution to Exercise 10.10.6 (p. 417)
C
Solution to Exercise 10.10.7 (p. 418)
$N\left(2, \frac{.25}{\sqrt{16}}\right)$
Solution to Exercise 10.10.9 (p. 418)
0.5000
Solution to Exercise 10.10.10 (p. 418)
7.6
Solution to Exercise 10.10.11 (p. 418)
5
Solution to Exercise 10.10.12 (p. 418)
0.9431