Chapter 8

Confidence Intervals

8.1 Confidence Intervals[1]

8.1.1 Student Learning Objectives

By the end of this chapter, the student should be able to:

- Calculate and interpret confidence intervals for one population average and one population proportion.
- Interpret the student-t probability distribution as the sample size changes.
- Discriminate between problems applying the normal and the student-t distributions.

8.1.2 Introduction

Suppose you are trying to determine the average rent of a two-bedroom apartment in your town. You might look in the classified section of the newspaper, write down several rents listed, and average them together. You would have obtained a point estimate of the true mean. If you are trying to determine the percent of times you make a basket when shooting a basketball, you might count the number of shots you make and divide that by the number of shots you attempted. In this case, you would have obtained a point estimate for the true proportion.

We use sample data to make generalizations about an unknown population. This part of statistics is called **inferential statistics**. **The sample data help us to make an estimate of a population parameter**. We realize that the point estimate is most likely not the exact value of the population parameter, but close to it. After calculating point estimates, we construct confidence intervals in which we believe the parameter lies.

In this chapter, you will learn to construct and interpret confidence intervals. You will also learn a new distribution, the Student-t, and how it is used with these intervals.

If you worked in the marketing department of an entertainment company, you might be interested in the average number of compact discs (CD's) a consumer buys per month. If so, you could conduct a survey and calculate the sample average, $\overline{x}$, and the sample standard deviation, s. You would use $\overline{x}$ to estimate the population mean and s to estimate the population standard deviation. The sample mean, $\overline{x}$, is the **point estimate** for the population mean, μ. The sample standard deviation, s, is the point estimate for the population standard deviation, σ.

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[1]This content is available online at <http://cnx.org/content/m16967/1.10/>.

A **confidence interval** is another type of estimate but, instead of being just one number, it is an interval of numbers. The interval of numbers is an estimated range of values calculated from a given set of sample data. The confidence interval is likely to include an unknown population parameter.

Suppose for the CD example we do not know the population mean μ but we do know that the population standard deviation is $\sigma = 1$ and our sample size is 100. Then by the Central Limit Theorem, the standard deviation for the sample mean is

$\frac{\sigma}{\sqrt{n}} = \frac{1}{\sqrt{100}} = 0.1$.

The **Empirical Rule**, which applies to bell-shaped distributions, says that in approximately 95% of the samples, the sample mean, $\overline{x}$, will be within two standard deviations of the population mean μ. For our CD example, two standard deviations is $(2)(0.1) = 0.2$. The sample mean $\overline{x}$ is within 0.2 units of μ.

Because $\overline{x}$ is within 0.2 units of μ, which is unknown, then μ is within 0.2 units of $\overline{x}$ in 95% of the samples. The population mean μ is contained in an interval whose lower number is calculated by taking the sample mean and subtracting two standard deviations $((2)(0.1))$ and whose upper number is calculated by taking the sample mean and adding two standard deviations. In other words, μ is between $\overline{x} - 0.2$ and $\overline{x} + 0.2$ in 95% of all the samples.

For the CD example, suppose that a sample produced a sample mean $\overline{x} = 2$. Then the unknown population mean μ is between

$\overline{x} - 0.2 = 2 - 0.2 = 1.8$ and $\overline{x} + 0.2 = 2 + 0.2 = 2.2$

We say that we are **95% confident** that the unknown population mean number of CDs is between 1.8 and 2.2. **The 95% confidence interval is (1.8, 2.2).**

The 95% confidence interval implies two possibilities. Either the interval (1.8, 2.2) contains the true mean μ or our sample produced an $\overline{x}$ that is not within 0.2 units of the true mean μ. The second possibility happens for only 5% of all the samples (100% - 95%).

Remember that a confidence interval is created for an unknown population parameter like the population mean, μ. A confidence interval has the form

(point estimate - margin of error, point estimate + margin of error)

The margin of error depends on the confidence level or percentage of confidence.

8.1.3 Optional Collaborative Classroom Activity

Have your instructor record the number of meals each student in your class eats out in a week. Assume that the standard deviation is known to be 3 meals. Construct an approximate 95% confidence interval for the true average number of meals students eat out each week.

1. Calculate the sample mean.
2. $\sigma = 3$ and $n =$ the number of students surveyed.
3. Construct the interval $\left(\overline{x} - 2 \cdot \frac{\sigma}{\sqrt{n}}, \overline{x} + 2 \cdot \frac{\sigma}{\sqrt{n}} \right)$

We say we are approximately 95% confident that the true average number of meals that students eat out in a week is between _____ and _____.

8.2 Confidence Interval, Single Population Mean, Population Standard Deviation Known, Normal[2]

To construct a confidence interval for a single unknown population mean μ **where the population standard deviation is known,** we need $\bar{x}$ as an estimate for μ and a margin of error. Here, the margin of error is called the **error bound for a population mean** (abbreviated **EBM**). The margin of error depends on the **confidence level** (abbreviated **CL**). The confidence level is the probability that the confidence interval produced contains the true population parameter. Most often, it is the choice of the person constructing the confidence interval to choose a confidence level of 90% or higher because he wants to be reasonably certain of his conclusions.

There is another probability called **alpha** (α). α is the probability that the sample produced a point estimate that is not within the appropriate margin of error of the unknown population parameter.

> **Example 8.1**
> Suppose the sample mean is 7 and the error bound for the mean is 2.5.
>
> **Problem** *(Solution on p. 327.)*
> $\bar{x} = $ _____ and $EBM = $ _____.
> The confidence interval is $(7 - 2.5, 7 + 2.5)$.

If the confidence level (CL) is 95%, then we say we are 95% confident that the true population mean is between 4.5 and 9.5.

A confidence interval for a population mean with a known standard deviation is based on the fact that the sample means follow an approximately normal distribution. Suppose we have constructed the 90% confidence interval (5, 15) where $\bar{x} = 10$ and $EBM = 5$. To get a 90% confidence interval, we must include the central 90% of the sample means. If we include the central 90%, we leave out a total of 10 % or 5% in each tail of the normal distribution. To capture the central 90% of the sample means, we must go out 1.645 standard deviations on either side of the calculated sample mean. The 1.645 is the z-score from a standard normal table that has area to the right equal to 0.05 (5% area in the right tail). The graph shows the general situation.

Confidence Level (CL) = 0.90
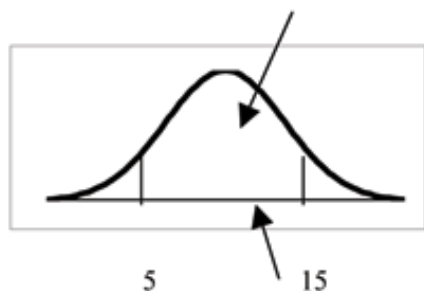
$\bar{x} = 10$

$EBM = 5$

$\bar{x} - EBM = 5$

$\bar{x} + EBM = 15$

μ is believed to be in the interval (5, 15) with 90% confidence.

To summarize, resulting from the Central Limit Theorem,

$\overline{X}$ is normally distributed, that is, $\overline{X} \sim N\left(\mu_X, \frac{\sigma_X}{\sqrt{n}}\right)$

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[2]This content is available online at <http://cnx.org/content/m16962/1.13/>.

Since the population standard deviation, σ, is known, we use a normal curve.

The confidence level, CL, is $CL = 1 - \alpha$. Each of the tails contains an area equal to $\frac{\alpha}{2}$.

The z-score that has area to the right of $\frac{\alpha}{2}$ is denoted by $z_{\frac{\alpha}{2}}$.

For example, if $\frac{\alpha}{2} = 0.025$, then area to the right $= 0.025$ and area to the left $= 1 - 0.025 = 0.975$ and $z_{\frac{\alpha}{2}} = z_{0.025} = 1.96$ using a calculator, computer or table. Using the TI83+ or 84 calculator function, invNorm, you can verify this result. invNorm$(.975, 0, 1) = 1.96$.
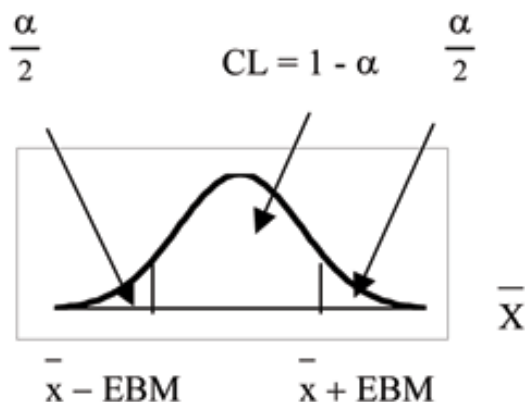
The error bound formula for a single population mean when the population standard deviation is known is

$EBM = z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}$

The confidence interval has the format $(\overline{x} - EBM, \overline{x} + EBM)$.

The graph gives a picture of the entire situation.

$CL + \frac{\alpha}{2} + \frac{\alpha}{2} = CL + \alpha = 1.$

Example 8.2
Problem 1
 Suppose scores on exams in statistics are normally distributed with an unknown population mean and a population standard deviation of 3 points. A sample of 36 scores is taken and gives a sample mean (sample average score) of 68. Find a 90% confidence interval for the true (population) mean of statistics exam scores.

- The first solution is step-by-step.
- The second solution uses the TI-83+ and TI-84 calculators.

Solution A
To find the confidence interval, you need the sample mean, $\overline{x}$, and the EBM.

a. $\overline{x} = 68$
b. $EBM = z_{\frac{\alpha}{2}} \cdot \left(\frac{\sigma}{\sqrt{n}} \right)$
c. $\sigma = 3$
d. $n = 36$

$CL = 0.90$ so $a = 1 - CL = 1 - 0.90 = 0.10$

Since $\frac{\alpha}{2} = 0.05$, then $z_{\frac{\alpha}{2}} = z_{.05} = 1.645$

from a calculator, computer or standard normal table. For the table, see the Table of Contents **15. Tables**.

Therefore, $EBM = 1.645 \cdot \left(\frac{3}{\sqrt{36}} \right) = 0.8225$

This gives $\overline{x} - EBM = 68 - 0.8225 = 67.18$

and $\overline{x} + EBM = 68 + 0.8225 = 68.82$

The 90% confidence interval is **(67.18, 68.82).**

Solution B
The TI-83+ and TI-84 caculators simplify this whole procedure. Press STAT and arrow over to TESTS. Arrow down to 7:ZInterval. Press ENTER. Arrow to Stats and press ENTER. Arrow down and enter 3 for σ, 68 for $\overline{x}$, 36 for n, and .90 for C-level. Arrow down to Calculate and press ENTER. The confidence interval is (to 3 decimal places) (67.178, 68.822).

We can find the error bound from the confidence interval. From the upper value, subtract the sample mean **or** subtract the lower value from the upper value and divide by two. The result is the error bound for the mean (EBM).

$EBM = 68.822 - 68 = 0.822$ or $EBM = \frac{(68.822 - 67.178)}{2} = 0.822$

We can interpret the confidence interval in two ways:

1. We are 90% confident that the true population mean for statistics exam scores is between 67.178 and 68.822.
2. Ninety percent of all confidence intervals constructed in this way contain the true average statistics exam score. For example, if we constructed 100 of these confidence intervals, we would expect 90 of them to contain the true population mean exam score.

Now for the same problem, find a 95% confidence interval for the true (population) mean of scores. Draw the graph. The sample mean, standard deviation, and sample size are:

Problem 2 *(Solution on p. 327.)*

    a. $\overline{x} =$
    b. $\sigma =$
    c. $n =$

The confidence level is $CL = 0.95$.   Graph:

The confidence interval is (use technology)

Problem 3
$(\overline{x} - EBM, \overline{x} + EBM) = ($_____ , _____$)$. The error bound $EBM =$ _____.

Solution
$(\overline{x} - EBM, \overline{x} + EBM) = $ *(67.02 , 68.98)*. The error bound $EBM = 0.98$.

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We can say that we are 95 % confident that the true population mean for statistics exam scores is between 67.02 and 68.98 and that 95% of all confidence intervals constructed in this way contain the true average statistics exam score.

Example 8.3
Suppose we change the previous problem.

Problem 1
Leave everything the same except the sample size. For this problem, we can examine the impact of changing n to 100 or changing n to 25.

   a. $\bar{x} = 68$
   b. $\sigma = 3$
   c. $z_{\frac{\alpha}{2}} = 1.645$

Solution A
If we **increase** the sample size n to 100, we **decrease** the error bound.

$$EBM = z_{\frac{\alpha}{2}} \cdot \left(\frac{\sigma}{\sqrt{n}}\right) = 1.645 \cdot \left(\frac{3}{\sqrt{100}}\right) = 0.4935$$

Solution B
If we **decrease** the sample size n to 25, we **increase** the error bound.

$$EBM = z_{\frac{\alpha}{2}} \cdot \left(\frac{\sigma}{\sqrt{n}}\right) = 1.645 \cdot \left(\frac{3}{\sqrt{25}}\right) = 0.987$$

Problem 2
Leave everything the same except for the confidence level. We increase the confidence level from 0.90 to 0.95.

   a. $\bar{x} = 68$
   b. $\sigma = 3$
   c. $z_{\frac{\alpha}{2}}$ changes from 1.645 to 1.96.

Solution

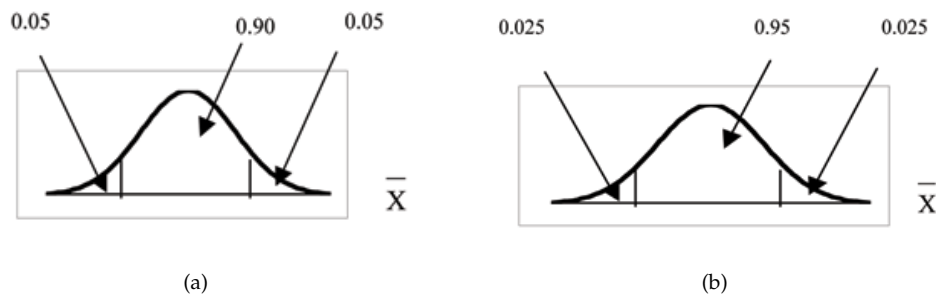(a)   (b)

Figure 8.1

The 90% confidence interval is (67.18, 68.82). The 95% confidence interval is (67.02, 68.98). The 95% confidence interval is wider. **If you look at the graphs, because the area 0.95 is larger than the area 0.90, it makes sense that the 95% confidence interval is wider.**

8.2.1

Calculating the Sample Size n

If researchers desire a specific margin of error, then they can use the error bound formula to calculate the required sample size.

The error bound formula for a population mean when the population standard deviation is known is

- $EBM = z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}$
- Solving for n gives you an equation for the sample size.
- $n = \frac{z_{\frac{\alpha}{2}}^2 \cdot \sigma^2}{EBM^2}$

   Example 8.4
   The population standard deviation for the age of Foothill College students is 15 years. If we want to be 95% confident that the sample mean age is within 2 years of the true population mean age of Foothill College students , how many randomly selected Foothill College students must be surveyed?

From the problem, we know that

- $\sigma = 15$
- $EBM = 2$
- $z_{\frac{\alpha}{2}} = 1.96$ because the confidence level is 95%.

Using the equation for the sample size, we have

- $n = \frac{z_{\frac{\alpha}{2}}^2 \cdot \sigma^2}{EBM^2}$
- $n = \frac{1.96^2 \cdot 15^2}{2^2}$
- $n = 216.09$
- Round the answer to the next higher value to ensure that the sample size is as large as it should be. Therefore, 217 Foothill College students should be surveyed for us to be 95% confident that we are within 2 years of the true population age of Foothill College students.

   NOTE: In reality, we usually do not know the population standard deviation so we estimate it with the sample standard deviation or use some other way of estimating it (for example, some statisticians use the results of some other earlier study as the estimate).

8.3 Confidence Interval, Single Population Mean, Standard Deviation Unknown, Student-T[3]

In practice, we rarely know the population **standard deviation**. In the past, when the sample size was large, this did not present a problem to statisticians. They used the sample standard deviation s as an estimate for σ and proceeded as before to calculate a **confidence interval** with close enough results. However, statisticians ran into problems when the sample size was small. A small sample size caused inaccuracies in the confidence interval. William S. Gossett of the Guinness brewery in Dublin, Ireland ran into this very problem. His experiments with hops and barley produced very few samples. Just replacing σ with s did not produce accurate results when he tried to calculate a confidence interval. He realized that he could not use a normal distribution for the calculation. This problem led him to "discover" what is called the **Student-t distribution**. The name comes from the fact that Gosset wrote under the pen name "Student."

Up until the mid 1990s, statisticians used the **normal distribution** approximation for large sample sizes and only used the Student-t distribution for sample sizes of at most 30. With the common use of graphing calculators and computers, the practice is to use the Student-t distribution whenever s is used as an estimate for σ.

If you draw a simple random sample of size n from a population that has approximately a normal distribution with mean μ and unknown population standard deviation σ and calculate the t-score $t = \frac{\bar{x} - \mu}{\left(\frac{s}{\sqrt{n}} \right)}$, then the t-scores follow a **Student-t distribution with** $n - 1$ **degrees of freedom**. The t-score has the same interpretation as the **z-score**. It measures how far $\bar{x}$ is from its mean μ. For each sample size n, there is a different Student-t distribution.

The **degrees of freedom**, $n - 1$, come from the sample standard deviation s. In Chapter 2, we used n deviations ($x - \bar{x}$ *values*) to calculate s. Because the sum of the deviations is 0, we can find the last deviation once we know the other $n - 1$ deviations. The other $n - 1$ deviations can change or vary freely. **We call the number** $n - 1$ **the degrees of freedom (df).**

The following are some facts about the Student-t distribution:

1. The graph for the Student-t distribution is similar to the normal curve.
2. The Student-t distribution has more probability in its tails than the normal because the spread is somewhat greater than the normal.
3. The underlying population of observations is normal with unknown population mean μ and unknown population standard deviation σ. In the real world, however, as long as the underlying population is large and bell-shaped, and the data are a simple random sample, practitioners often consider the assumptions met.

A Student-t table (See the Table of Contents **15. Tables**) gives t-scores given the degrees of freedom and the right-tailed probability. The table is very limited. **Calculators and computers can easily calculate any Student-t probabilities.**

The notation for the Student-t distribution is (using T as the random variable)

$T \sim t_{df}$ where $df = n - 1$.

If the population standard deviation is **not known**, then the **error bound for a population mean** formula is:

$EBM = t_{\frac{\alpha}{2}} \cdot \left(\frac{s}{\sqrt{n}} \right)$ \qquad $t_{\frac{\alpha}{2}}$ is the t-score with area to the right equal to $\frac{\alpha}{2}$.

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[3]This content is available online at <http://cnx.org/content/m16959/1.11/>.

s = the sample standard deviation

The mechanics for calculating the error bound and the confidence interval are the same as when σ is known.

> **Example 8.5**
> Suppose you do a study of acupuncture to determine how effective it is in relieving pain. You measure sensory rates for 15 subjects with the results given below. Use the sample data to construct a 95% confidence interval for the mean sensory rate for the population (assumed normal) from which you took the data.
>
> 8.6; 9.4; 7.9; 6.8; 8.3; 7.3; 9.2; 9.6; 8.7; 11.4; 10.3; 5.4; 8.1; 5.5; 6.9
>
> **Note:**
>
> - The first solution is step-by-step.
> - The second solution uses the TI-83+ and TI-84 calculators.
>
> **Solution A**
> To find the confidence interval, you need the sample mean, $\overline{x}$, and the EBM.
>
> $\overline{x} = 8.2267 \qquad s = 1.6722 \qquad n = 15$
>
> $CL = 0.95 \quad$ so $\quad \alpha = 1 - CL = 1 - 0.95 = 0.05$
>
> $EBM = t_{\frac{\alpha}{2}} \cdot \left(\frac{s}{\sqrt{n}} \right)$
>
> $\frac{\alpha}{2} = 0.025 \qquad t_{\frac{\alpha}{2}} = t_{.025} = 2.14$
>
> (Student-t table with $df = 15 - 1 = 14$)
>
> Therefore, $EBM = 2.14 \cdot \left(\frac{1.6722}{\sqrt{15}} \right) = 0.924$
>
> This gives $\overline{x} - EBM = 8.2267 - 0.9240 = 7.3$
>
> and $\overline{x} + EBM = 8.2267 + 0.9240 = 9.15$
>
> The 95% confidence interval is **(7.30, 9.15)**.
>
> You are 95% confident or sure that the true population average sensory rate is between 7.30 and 9.15.
>
> **Solution B**
> TI-83+ or TI-84: Use the function `8:TInterval` in `STAT TESTS`. Once you are in `TESTS`, press `8:TInterval` and arrow to `Data`. Press `ENTER`. Arrow down and enter the list name where you put the data for `List`, enter 1 for `Freq`, and enter .95 for `C-level`. Arrow down to `Calculate` and press `ENTER`. The confidence interval is (7.3006, 9.1527)

8.4 Confidence Interval for a Population Proportion[4]

During an election year, we see articles in the newspaper that state **confidence intervals** in terms of proportions or percentages. For example, a poll for a particular candidate running for president might show

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[4]This content is available online at <http://cnx.org/content/m16963/1.10/>.

that the candidate has 40% of the vote within 3 percentage points. Often, election polls are calculated with 95% confidence. So, the pollsters would be 95% confident that the true proportion of voters who favored the candidate would be between 0.37 and 0.43 $(0.40 - 0.03, 0.40 + 0.03)$.

Investors in the stock market are interested in the true proportion of stocks that go up and down each week. Businesses that sell personal computers are interested in the proportion of households in the United States that own personal computers. Confidence intervals can be calculated for the true proportion of stocks that go up or down each week and for the true proportion of households in the United States that own personal computers.

The procedure to find the confidence interval, the sample size, the **error bound,** and the **confidence level** for a proportion is similar to that for the population mean. The formulas are different.

How do you know you are dealing with a proportion problem? First, the underlying **distribution is binomial**. (There is no mention of a mean or average.) If X is a binomial random variable, then $X \sim B(n, p)$ where n = the number of trials and p = the probability of a success. To form a proportion, take X, the random variable for the number of successes and divide it by n, the number of trials (or the sample size). The random variable P' (read "P prime") is that proportion,

$P' = \frac{X}{n}$

(Sometimes the random variable is denoted as $\hat{P}$, read "P hat".)

When n is large, we can use the **normal distribution** to approximate the binomial.

$X \sim N\left(n \cdot p, \sqrt{n \cdot p \cdot q}\right)$

If we divide the random variable by n, the mean by n, and the standard deviation by n, we get a normal distribution of proportions with P', called the estimated proportion, as the random variable. (Recall that a proportion = the number of successes divided by n.)

$\frac{X}{n} = P' \sim N\left(\frac{n \cdot p}{n}, \frac{\sqrt{n \cdot p \cdot q}}{n}\right)$

By algebra, $\frac{\sqrt{n \cdot p \cdot q}}{n} = \sqrt{\frac{p \cdot q}{n}}$

P' **follows a normal distribution for proportions**: $P' \sim N\left(p, \sqrt{\frac{p \cdot q}{n}}\right)$

The confidence interval has the form $(p' - EBP, p' + EBP)$.

$p' = \frac{x}{n}$

p' = the **estimated proportion** of successes (p' is a **point estimate** for p, the true proportion)

x = the **number** of successes.

n = the size of the sample

The error bound for a proportion is

$EBP = z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{p' \cdot q'}{n}} \qquad q' = 1 - p'$

This formula is actually very similar to the error bound formula for a mean. The difference is the standard deviation. For a mean where the population standard deviation is known, the standard deviation is $\frac{\sigma}{\sqrt{n}}$.

For a proportion, the standard deviation is $\sqrt{\frac{p \cdot q}{n}}$.

However, in the error bound formula, the standard deviation is $\sqrt{\frac{p' \cdot q'}{n}}$.

In the error bound formula, **p' and q' are estimates of** p **and** q. The estimated proportions p' and q' are used because p and q are not known. p' and q' are calculated from the data. p' is the estimated proportion of successes. q' is the estimated proportion of failures.

NOTE: For the normal distribution of proportions, the z-score formula is as follows.

If $P' \sim N\left(p, \sqrt{\frac{p \cdot q}{n}}\right)$ then the z-score formula is $z = \frac{p' - p}{\sqrt{\frac{p \cdot q}{n}}}$

Example 8.6

Suppose that a sample of 500 households in Phoenix was taken last May to determine whether the oldest child had given his/her mother a Mother's Day card. Of the 500 households, 421 responded yes. Compute a 95% confidence interval for the true proportion of all Phoenix households whose oldest child gave his/her mother a Mother's Day card.

Note:

- The first solution is step-by-step.
- The second solution uses the TI-83+ and TI-84 calculators.

Solution A

Let X = the number of oldest children who gave their mothers Mother's Day card last May. X is binomial. $X \sim B\left(500, \frac{421}{500}\right)$.

To calculate the confidence interval, you must find p', q', and *EBP*.

$n = 500 \qquad x$ = the number of successes = 421

$p' = \frac{x}{n} = \frac{421}{500} = 0.842$

$q' = 1 - p' = 1 - 0.842 = 0.158$

Since $CL = 0.95$, then $\alpha = 1 - CL = 1 - 0.95 = 0.05 \qquad \frac{\alpha}{2} = 0.025$.

Then $z_{\frac{\alpha}{2}} = z_{.025} = 1.96$ using a calculator, computer, or standard normal table.

Remember that the area to the right = 0.025 and therefore, area to the left is 0.975.

The z-score that corresponds to 0.975 is 1.96.

$EBP = z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{p' \cdot q'}{n}} = 1.96 \cdot \sqrt{\left[\frac{(.842) \cdot (.158)}{500}\right]} = 0.032$

$p' - EBP = 0.842 - 0.032 = 0.81$

$p' + EBP = 0.842 + 0.032 = 0.874$

The confidence interval for the true binomial population proportion is $(p' - EBP, p' + EBP) = (0.810, 0.874)$.

We are 95% confident that between 81% and 87.4% of the oldest children in households in Phoenix gave their mothers a Mother's Day card last May.

We can also say that 95% of the confidence intervals constructed in this way contain the true proportion of oldest children in Phoenix who gave their mothers a Mother's Day card last May.

Solution B

TI-83+ and TI-84: Press STAT and arrow over to TESTS. Arrow down to A:PropZint. Press ENTER. Enter 421 for x, 500 for n, and .95 for C-Level. Arrow down to Calculate and press ENTER. The confidence interval is (0.81003, 0.87397).

Example 8.7

For a class project, a political science student at a large university wants to determine the percent of students that are registered voters. He surveys 500 students and finds that 300 are registered voters. Compute a 90% confidence interval for the true percent of students that are registered voters and interpret the confidence interval.

Solution

$x = 300$ and $n = 500$. Using a TI-83+ or 84 calculator, the 90% confidence interval for the true percent of students that are registered voters is (0.564, 0.636).

Interpretation:

- We are 90% confident that the true percent of students that are registered voters is between 56.4% and 63.6%.
- Ninety percent (90 %) of all confidence intervals constructed in this way contain the true percent of students that are registered voters.

8.4.1

Calculating the Sample Size n

If researchers desire a specific margin of error, then they can use the error bound formula to calculate the required sample size.

The error bound formula for a population proportion is

- $EBM = z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{p'q'}{n}}$
- Solving for n gives you an equation for the sample size.
- $n = \frac{z_{\frac{\alpha}{2}}^2 \cdot p'q'}{EBM^2}$

Example 8.8

Suppose a mobile phone company wants to determine the current percentage of customers aged 50+ that use text messaging on their cell phone. How many customers aged 50+ should the company survey in order to be 90% confident that the estimated (sample) proportion is within 3 percentage points of the true population proportion of customers aged 50+ that use text messaging on their cell phone.

From the problem, we know that

- $EBP = 0.03$ (3% $= 0.03$)

- $z_{\frac{\alpha}{2}} = 1.645$ because the confidence level is 90%.

However, in order to find n , we need to know the estimated (sample) proportion p'. Remember that $q' = 1 - p'$. But, we do not know p'. Since we multiply p' and q' together, we make them both equal to 0.5 because $p'q' = (.5)(.5) = 25$ results in the largest possible product. (Try other products: $(.6)(.4) = 24$; $(.3)(.7) = 21$; $(.2)(.8) = 16$; and so on). The largest possible product gives us the largest n. This gives us a large enough sample so that we can be 90% confident that we are within 3 percentage points of the true proportion of customers aged 50+ that use text messaging on their cell phone. To calculate the sample size n, use the formula and make the substitutions.

- $n = \dfrac{z_{\frac{\alpha}{2}}^{2} \cdot p'q'}{EBM^2}$
- $n = \dfrac{\left(1.645^2\right) \cdot (.5)(.5)}{.03^2}$
- $n = 751.7$
- Round the answer to the next higher value. The sample size should be 758 cell phone customers aged 50+ in order to be 90% confident that the estimated (sample) proportion is within 3 percentage points of the true population proportion of customers aged 50+ that use text messaging on their cell phone.

8.5 Summary of Formulas[5]

Formula 8.1: General form of a confidence interval

$(lower\ value, upper\ value) = (point\ estimate - error\ bound, point\ estimate + error\ bound)$

Formula 8.2: To find the error bound when you know the confidence interval

$error\ bound = upper\ value - point\ estimate$ OR $error\ bound = \frac{upper\ value - lower\ value}{2}$

Formula 8.3: Single Population Mean, Known Standard Deviation, Normal Distribution

Use the Normal Distribution for Means (Section 7.2) $EBM = z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}$

The confidence interval has the format $(\overline{x} - EBM, \overline{x} + EBM)$.

Formula 8.4: Single Population Mean, Unknown Standard Deviation, Student-t Distribution

Use the Student-t Distribution with degrees of freedom $df = n - 1$. $EBM = t_{\frac{\alpha}{2}} \cdot \frac{s}{\sqrt{n}}$

Formula 8.5: Single Population Proportion, Normal Distribution

Use the Normal Distribution for a single population proportion $p' = \frac{x}{n}$

$EBP = z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{p' \cdot q'}{n}}$ $p' + q' = 1$

The confidence interval has the format $(p' - EBP, p' + EBP)$.

Formula 8.6: Point Estimates

$\overline{x}$ is a point estimate for μ

p' is a point estimate for ρ

s is a point estimate for σ

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[5]This content is available online at <http://cnx.org/content/m16973/1.7/>.

8.6 Practice 1: Confidence Intervals for Averages, Known Population Standard Deviation[6]

8.6.1 Student Learning Outcomes

- The student will explore the properties of Confidence Intervals for Averages when the population standard deviation is known.

8.6.2 Given

The average age for all Foothill College students for Fall 2005 was 32.7. The population standard deviation has been pretty consistent at 15. Twenty-five Winter 2006 students were randomly selected. The average age for the sample was 30.4. We are interested in the true average age for Winter 2006 Foothill College students. (http://research.fhda.edu/factbook/FHdemofs/Fact_sheet_fh_2005f.pdf[7])

Let X = the age of a Winter 2006 Foothill College student

8.6.3 Calculating the Confidence Interval

Exercise 8.6.1   *(Solution on p. 327.)*
$\overline{x} =$

Exercise 8.6.2   *(Solution on p. 327.)*
$n=$

Exercise 8.6.3   *(Solution on p. 327.)*
15=(insert symbol here)

Exercise 8.6.4   *(Solution on p. 327.)*
Define the Random Variable, $\overline{X}$, in words.

$\overline{X} =$

Exercise 8.6.5   *(Solution on p. 327.)*
What is $\overline{x}$ estimating?

Exercise 8.6.6   *(Solution on p. 327.)*
Is σ_x known?

Exercise 8.6.7   *(Solution on p. 327.)*
As a result of your answer to (4), state the exact distribution to use when calculating the Confidence Interval.

8.6.4 Explaining the Confidence Interval

Construct a 95% Confidence Interval for the true average age of Winter 2006 Foothill College students.

Exercise 8.6.8   *(Solution on p. 327.)*
How much area is in both tails (combined)? $\alpha =$ _____

Exercise 8.6.9   *(Solution on p. 327.)*
How much area is in each tail? $\frac{\alpha}{2} =$ _____

Exercise 8.6.10   *(Solution on p. 327.)*
Identify the following specifications:

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[6]This content is available online at <http://cnx.org/content/m16970/1.9/>.
[7]http://research.fhda.edu/factbook/FHdemofs/Fact_sheet_fh_2005f.pdf

a. lower limit =
b. upper limit =
c. error bound =

Exercise 8.6.11   *(Solution on p. 327.)*
The 95% Confidence Interval is:_____

Exercise 8.6.12

Fill in the blanks on the graph with the areas, upper and lower limits of the Confidence Interval, and the sample mean.

$$\frac{\alpha}{2} = \underline{\hspace{1cm}} \qquad \text{C.L.} = \underline{\hspace{1cm}} \qquad \frac{\alpha}{2} = \underline{\hspace{1cm}}$$
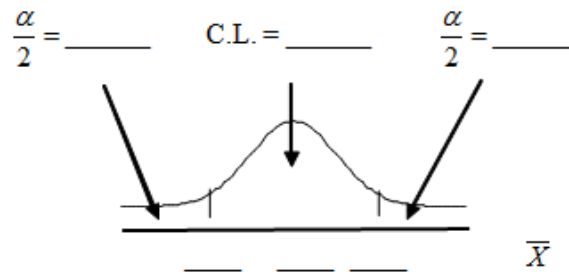
Figure 8.2

Exercise 8.6.13
In one complete sentence, explain what the interval means.

8.6.5 Discussion Questions

Exercise 8.6.14
Using the same mean, standard deviation and level of confidence, suppose that n were 69 instead of 25. Would the error bound become larger or smaller? How do you know?

Exercise 8.6.15
Using the same mean, standard deviation and sample size, how would the error bound change if the confidence level were reduced to 90%? Why?

8.7 Practice 2: Confidence Intervals for Averages, Unknown Population Standard Deviation[8]

8.7.1 Student Learning Outcomes

- The student will explore the properties of confidence intervals for averages, as well as the properties of an unknown population standard deviation.

8.7.2 Given

The following real data are the result of a random survey of 39 national flags (with replacement between picks) from various countries. We are interested in finding a confidence interval for the true average number of colors on a national flag. Let X = the number of colors on a national flag.

X	Freq.
1	1
2	7
3	18
4	7
5	6

Table 8.1

8.7.3 Calculating the Confidence Interval

Exercise 8.7.1   *(Solution on p. 327.)*
Calculate the following:

   a. $\overline{x} =$
   b. $s_x =$
   c. $n =$

Exercise 8.7.2   *(Solution on p. 327.)*
Define the Random Variable, $\overline{X}$, in words. $\overline{X} =$ _____

Exercise 8.7.3   *(Solution on p. 327.)*
What is $\overline{x}$ estimating?

Exercise 8.7.4   *(Solution on p. 327.)*
Is σ_x known?

Exercise 8.7.5   *(Solution on p. 328.)*
As a result of your answer to (4), state the exact distribution to use when calculating the Confidence Interval.

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[8]This content is available online at <http://cnx.org/content/m16971/1.10/>.

8.7.4 Confidence Interval for the True Average Number

Construct a 95% Confidence Interval for the true average number of colors on national flags.

Exercise 8.7.6   *(Solution on p. 328.)*
How much area is in both tails (combined)? $\alpha =$

Exercise 8.7.7   *(Solution on p. 328.)*
How much area is in each tail? $\frac{\alpha}{2} =$

Exercise 8.7.8   *(Solution on p. 328.)*
Calculate the following:

   a. lower limit =
   b. upper limit =
   c. error bound =

Exercise 8.7.9   *(Solution on p. 328.)*
The 95% Confidence Interval is:

Exercise 8.7.10
Fill in the blanks on the graph with the areas, upper and lower limits of the Confidence Interval, and the sample mean.

$$\frac{\alpha}{2} = \underline{\quad\quad} \qquad C.L. = \underline{\quad\quad} \qquad \frac{\alpha}{2} = \underline{\quad\quad}$$
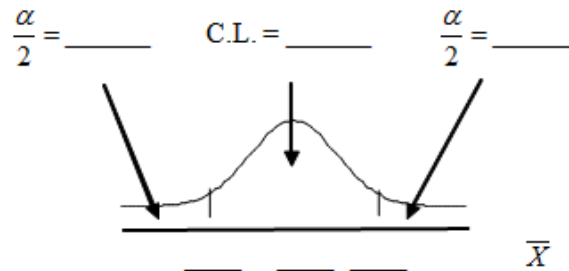
Figure 8.3

Exercise 8.7.11
In one complete sentence, explain what the interval means.

8.7.5 Discussion Questions

Exercise 8.7.12
Using the same $\bar{x}$, s_x, and level of confidence, suppose that n were 69 instead of 39. Would the error bound become larger or smaller? How do you know?

Exercise 8.7.13
Using the same $\bar{x}$, s_x, and $n = 39$, how would the error bound change if the confidence level were reduced to 90%? Why?

8.8 Practice 3: Confidence Intervals for Proportions[9]

8.8.1 Student Learning Outcomes

- The student will explore the properties of the confidence intervals for proportions.

8.8.2 Given

The Ice Chalet offers dozens of different beginning ice-skating classes. All of the class names are put into a bucket. The 5 P.M., Monday night, ages 8 - 12, beginning ice-skating class was picked. In that class were 64 girls and 16 boys. Suppose that we are interested in the true proportion of girls, ages 8 - 12, in all beginning ice-skating classes at the Ice Chalet.

8.8.3 Estimated Distribution

Exercise 8.8.1
What is being counted?

Exercise 8.8.2 *(Solution on p. 328.)*
In words, define the Random Variable X. $X =$

Exercise 8.8.3 *(Solution on p. 328.)*
Calculate the following:

    a. $x =$
    b. $n =$
    c. $p' =$

Exercise 8.8.4 *(Solution on p. 328.)*
State the estimated distribution of X. $X \sim$

Exercise 8.8.5 *(Solution on p. 328.)*
Define a new Random Variable P'. What is p' estimating?

Exercise 8.8.6 *(Solution on p. 328.)*
In words, define the Random Variable P' . $P' =$

Exercise 8.8.7
State the estimated distribution of P'. $P' \sim$

8.8.4 Explaining the Confidence Interval

Construct a 92% Confidence Interval for the true proportion of girls in the age 8 - 12 beginning ice-skating classes at the Ice Chalet.

Exercise 8.8.8 *(Solution on p. 328.)*
How much area is in both tails (combined)? $\alpha =$

Exercise 8.8.9 *(Solution on p. 328.)*
How much area is in each tail? $\frac{\alpha}{2} =$

Exercise 8.8.10 *(Solution on p. 328.)*
Calculate the following:

    a. lower limit =

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[9]This content is available online at <http://cnx.org/content/m16968/1.10/>.

b. upper limit =
c. error bound =

Exercise 8.8.11   *(Solution on p. 328.)*
The 92% Confidence Interval is:

Exercise 8.8.12

Fill in the blanks on the graph with the areas, upper and lower limits of the Confidence Interval, and the sample proportion.

$$\frac{\alpha}{2} = \underline{\hspace{1cm}} \qquad C.L. = \underline{\hspace{1cm}} \qquad \frac{\alpha}{2} = \underline{\hspace{1cm}}$$

P'
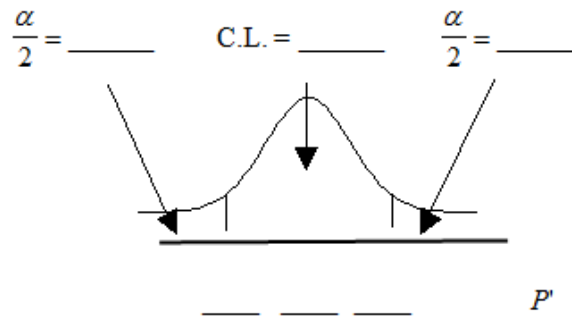
Figure 8.4

Exercise 8.8.13
In one complete sentence, explain what the interval means.

8.8.5 Discussion Questions

Exercise 8.8.14
Using the same p' and level of confidence, suppose that n were increased to 100. Would the error bound become larger or smaller? How do you know?

Exercise 8.8.15
Using the same p' and $n = 80$, how would the error bound change if the confidence level were increased to 98%? Why?

Exercise 8.8.16
If you decreased the allowable error bound, why would the minimum sample size increase (keeping the same level of confidence)?

8.9 Homework[10]

NOTE: If you are using a student-t distribution for a homework problem below, you may assume that the underlying population is normally distributed. (In general, you must first prove that assumption, though.)

Exercise 8.9.1   *(Solution on p. 328.)*
Among various ethnic groups, the standard deviation of heights is known to be approximately 3 inches. We wish to construct a 95% confidence interval for the average height of male Swedes. 48 male Swedes are surveyed. The sample mean is 71 inches. The sample standard deviation is 2.8 inches.

a. i. $\bar{x} =$_____
   ii. $\sigma =$_____
   iii. $s_x =$_____
   iv. $n =$_____
   v. $n - 1 =$_____
b. Define the Random Variables X and $\overline{X}$, in words.
c. Which distribution should you use for this problem? Explain your choice.
d. Construct a 95% confidence interval for the population average height of male Swedes.

   i. State the confidence interval.
   ii. Sketch the graph.
   iii. Calculate the error bound.

e. What will happen to the level of confidence obtained if 1000 male Swedes are surveyed instead of 48? Why?

Exercise 8.9.2
In six packages of "The Flintstones® Real Fruit Snacks" there were 5 Bam-Bam snack pieces. The total number of snack pieces in the six bags was 68. We wish to calculate a 96% confidence interval for the population proportion of Bam-Bam snack pieces.

a. Define the Random Variables X and P', in words.
b. Which distribution should you use for this problem? Explain your choice
c. Calculate p'.
d. Construct a 96% confidence interval for the population proportion of Bam-Bam snack pieces per bag.

   i. State the confidence interval.
   ii. Sketch the graph.
   iii. Calculate the error bound.

e. Do you think that six packages of fruit snacks yield enough data to give accurate results? Why or why not?

Exercise 8.9.3   *(Solution on p. 329.)*
A random survey of enrollment at 35 community colleges across the United States yielded the following figures (source: **Microsoft Bookshelf**): 6414; 1550; 2109; 9350; 21828; 4300; 5944; 5722; 2825; 2044; 5481; 5200; 5853; 2750; 10012; 6357; 27000; 9414; 7681; 3200; 17500; 9200; 7380; 18314; 6557; 13713; 17768; 7493; 2771; 2861; 1263; 7285; 28165; 5080; 11622. Assume the underlying population is normal.

a. i. $\bar{x} =$

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[10]This content is available online at <http://cnx.org/content/m16966/1.11/>.

      ii. $s_x =$ _____
      iii. $n =$ _____
      iv. $n - 1 =$ _____

b. Define the Random Variables X and $\overline{X}$, in words.

c. Which distribution should you use for this problem? Explain your choice.

d. Construct a 95% confidence interval for the population average enrollment at community colleges in the United States.

      i. State the confidence interval.
      ii. Sketch the graph.
      iii. Calculate the error bound.

e. What will happen to the error bound and confidence interval if 500 community colleges were surveyed? Why?

Exercise 8.9.4

From a stack of **IEEE Spectrum** magazines, announcements for 84 upcoming engineering conferences were randomly picked. The average length of the conferences was 3.94 days, with a standard deviation of 1.28 days. Assume the underlying population is normal.

a. Define the Random Variables X and $\overline{X}$, in words.

b. Which distribution should you use for this problem? Explain your choice.

c. Construct a 95% confidence interval for the population average length of engineering conferences.

      i. State the confidence interval.
      ii. Sketch the graph.
      iii. Calculate the error bound.

Exercise 8.9.5   *(Solution on p. 329.)*

Suppose that a committee is studying whether or not there is waste of time in our judicial system. It is interested in the average amount of time individuals waste at the courthouse waiting to be called for service. The committee randomly surveyed 81 people. The sample average was 8 hours with a sample standard deviation of 4 hours.

    a. i. $\overline{x} =$ _____
       ii. $s_x =$ _____
       iii. $n =$ _____
       iv. $n - 1 =$ _____

b. Define the Random Variables X and $\overline{X}$, in words.

c. Which distribution should you use for this problem? Explain your choice.

d. Construct a 95% confidence interval for the population average time wasted.

      a. State the confidence interval.
      b. Sketch the graph.
      c. Calculate the error bound.

e. Explain in a complete sentence what the confidence interval means.

Exercise 8.9.6

Suppose that an accounting firm does a study to determine the time needed to complete one person's tax forms. It randomly surveys 100 people. The sample average is 23.6 hours. There is a known standard deviation of 7.0 hours. The population distribution is assumed to be normal.

    a. i. $\overline{x} =$ _____
       ii. $\sigma =$ _____

    iii. $s_x =$_____

    iv. $n =$_____

    v. $n - 1 =$_____

b. Define the Random Variables X and $\overline{X}$, in words.

c. Which distribution should you use for this problem? Explain your choice.

d. Construct a 90% confidence interval for the population average time to complete the tax forms.

    i. State the confidence interval.

    ii. Sketch the graph.

    iii. Calculate the error bound.

e. If the firm wished to increase its level of confidence and keep the error bound the same by taking another survey, what changes should it make?

f. If the firm did another survey, kept the error bound the same, and only surveyed 49 people, what would happen to the level of confidence? Why?

g. Suppose that the firm decided that it needed to be at least 96% confident of the population average length of time to within 1 hour. How would the number of people the firm surveys change? Why?

Exercise 8.9.7   **(Solution on p. 329.)**

A sample of 16 small bags of the same brand of candies was selected. Assume that the population distribution of bag weights is normal. The weight of each bag was then recorded. The mean weight was 2 ounces with a standard deviation of 0.12 ounces. The population standard deviation is known to be 0.1 ounce.

a. i. $\overline{x} =$_____

    ii. $\sigma =$_____

    iii. $s_x =$_____

    iv. $n =$_____

    v. $n - 1 =$_____

b. Define the Random Variable X, in words.

c. Define the Random Variable $\overline{X}$, in words.

d. Which distribution should you use for this problem? Explain your choice.

e. Construct a 90% confidence interval for the population average weight of the candies.

    i. State the confidence interval.

    ii. Sketch the graph.

    iii. Calculate the error bound.

f. Construct a 98% confidence interval for the population average weight of the candies.

    i. State the confidence interval.

    ii. Sketch the graph.

    iii. Calculate the error bound.

g. In complete sentences, explain why the confidence interval in (f) is larger than the confidence interval in (e).

h. In complete sentences, give an interpretation of what the interval in (f) means.

Exercise 8.9.8

A pharmaceutical company makes tranquilizers. It is assumed that the distribution for the length of time they last is approximately normal. Researchers in a hospital used the drug on a random sample of 9 patients. The effective period of the tranquilizer for each patient (in hours) was as follows: 2.7; 2.8; 3.0; 2.3; 2.3; 2.2; 2.8; 2.1; and 2.4 .

a. i. $\overline{x} =$ _____

    ii. $s_x =$ _____

    iii. $n =$ _____

    iv. $n - 1 =$ _____

b. Define the Random Variable X, in words.

c. Define the Random Variable $\overline{X}$, in words.

d. Which distribution should you use for this problem? Explain your choice.

e. Construct a 95% confidence interval for the population average length of time.

    i. State the confidence interval.

    ii. Sketch the graph.

    iii. Calculate the error bound.

f. What does it mean to be "95% confident" in this problem?

Exercise 8.9.9   *(Solution on p. 329.)*

Suppose that 14 children were surveyed to determine how long they had to use training wheels. It was revealed that they used them an average of 6 months with a sample standard deviation of 3 months. Assume that the underlying population distribution is normal.

a. i. $\overline{x} =$ _____

    ii. $s_x =$ _____

    iii. $n =$ _____

    iv. $n - 1 =$ _____

b. Define the Random Variable X, in words.

c. Define the Random Variable $\overline{X}$, in words.

d. Which distribution should you use for this problem? Explain your choice.

e. Construct a 99% confidence interval for the population average length of time using train-
ing wheels.

    i. State the confidence interval.

    ii. Sketch the graph.

    iii. Calculate the error bound.

f. Why would the error bound change if the confidence level was lowered to 90%?

Exercise 8.9.10

Insurance companies are interested in knowing the population percent of drivers who always buckle up before riding in a car.

a. When designing a study to determine this population proportion, what is the minimum
number you would need to survey to be 95% confident that the population proportion
is estimated to within 0.03?

b. If it was later determined that it was important to be more than 95% confident and a new
survey was commissioned, how would that affect the minimum number you would
need to survey? Why?

Exercise 8.9.11   *(Solution on p. 330.)*

Suppose that the insurance companies did do a survey. They randomly surveyed 400 drivers and found that 320 claimed to always buckle up. We are interested in the population proportion of drivers who claim to always buckle up.

a. i. $x =$ _____

    ii. $n =$ _____

    iii. $p' =$ _____

b. Define the Random Variables X and P', in words.
c. Which distribution should you use for this problem? Explain your choice.
d. Construct a 95% confidence interval for the population proportion that claim to always buckle up.

   i. State the confidence interval.
   ii. Sketch the graph.
   iii. Calculate the error bound.

e. If this survey were done by telephone, list 3 difficulties the companies might have in obtaining random results.

Exercise 8.9.12
Unoccupied seats on flights cause airlines to lose revenue. Suppose a large airline wants to estimate its average number of unoccupied seats per flight over the past year. To accomplish this, the records of 225 flights are randomly selected and the number of unoccupied seats is noted for each of the sampled flights. The sample mean is 11.6 seats and the sample standard deviation is 4.1 seats.

a. i. $\bar{x} =$ _____
   ii. $s_x =$ _____
   iii. $n =$ _____
   iv. $n - 1 =$ _____
b. Define the Random Variables X and $\bar{X}$, in words.
c. Which distribution should you use for this problem? Explain your choice.
d. Construct a 92% confidence interval for the population average number of unoccupied seats per flight.

   i. State the confidence interval.
   ii. Sketch the graph.
   iii. Calculate the error bound.

Exercise 8.9.13   *(Solution on p. 330.)*
According to a recent survey of 1200 people, 61% feel that the president is doing an acceptable job. We are interested in the population proportion of people who feel the president is doing an acceptable job.

a. Define the Random Variables X and P', in words.
b. Which distribution should you use for this problem? Explain your choice.
c. Construct a 90% confidence interval for the population proportion of people who feel the president is doing an acceptable job.

   i. State the confidence interval.
   ii. Sketch the graph.
   iii. Calculate the error bound.

Exercise 8.9.14
A survey of the average amount of cents off that coupons give was done by randomly surveying one coupon per page from the coupon sections of a recent San Jose Mercury News. The following data were collected: 20¢; 75¢; 50¢; 65¢; 30¢; 55¢; 40¢; 40¢; 30¢; 55¢; $1.50; 40¢; 65¢; 40¢. Assume the underlying distribution is approximately normal.

a. i. $\bar{x} =$ _____
   ii. $s_x =$ _____
   iii. $n =$ _____

      iv. $n - 1 =$ _____

b. Define the Random Variables X and $\overline{X}$, in words.

c. Which distribution should you use for this problem? Explain your choice.

d. Construct a 95% confidence interval for the population average worth of coupons.

    i. State the confidence interval.

    ii. Sketch the graph.

    iii. Calculate the error bound.

e. If many random samples were taken of size 14, what percent of the confident intervals constructed should contain the population average worth of coupons? Explain why.

Exercise 8.9.15   *(Solution on p. 330.)*

An article regarding interracial dating and marriage recently appeared in the **Washington Post**. Of the 1709 randomly selected adults, 315 identified themselves as Latinos, 323 identified themselves as blacks, 254 identified themselves as Asians, and 779 identified themselves as whites. In this survey, 86% of blacks said that their families would welcome a white person into their families. Among Asians, 77% would welcome a white person into their families, 71% would welcome a Latino, and 66% would welcome a black person.

a. We are interested in finding the 95% confidence interval for the percent of black families that would welcome a white person into their families. Define the Random Variables X and P', in words.

b. Which distribution should you use for this problem? Explain your choice.

c. Construct a 95% confidence interval

    i. State the confidence interval.

    ii. Sketch the graph.

    iii. Calculate the error bound.

Exercise 8.9.16

Refer to the problem above.

a. Construct the 95% confidence intervals for the three Asian responses.

b. Even though the three point estimates are different, do any of the confidence intervals overlap? Which?

c. For any intervals that do overlap, in words, what does this imply about the significance of the differences in the true proportions?

d. For any intervals that do not overlap, in words, what does this imply about the significance of the differences in the true proportions?

Exercise 8.9.17   *(Solution on p. 330.)*

A camp director is interested in the average number of letters each child sends during his/her camp session. The population standard deviation is known to be 2.5. A survey of 20 campers is taken. The average from the sample is 7.9 with a sample standard deviation of 2.8.

a. **i.** $\overline{x} =$ _____

    ii. $\sigma =$ _____

    iii. $s_x =$ _____

    iv. $n =$ _____

    v. $n - 1 =$ _____

b. Define the Random Variables X and $\overline{X}$, in words.

c. Which distribution should you use for this problem? Explain your choice.

d. Construct a 90% confidence interval for the population average number of letters campers send home.

     i. State the confidence interval.
     ii. Sketch the graph.
     iii. Calculate the error bound.

  e. What will happen to the error bound and confidence interval if 500 campers are surveyed? Why?

Exercise 8.9.18

Stanford University conducted a study of whether running is healthy for men and women over age 50. During the first eight years of the study, 1.5% of the 451 members of the 50-Plus Fitness Association died. We are interested in the proportion of people over 50 who ran and died in the same eight–year period.

  a. Define the Random Variables X and P', in words.
  b. Which distribution should you use for this problem? Explain your choice.
  c. Construct a 97% confidence interval for the population proportion of people over 50 who ran and died in the same eight–year period.

     i. State the confidence interval.
     ii. Sketch the graph.
     iii. Calculate the error bound.

  d. Explain what a "97% confidence interval" means for this study.

Exercise 8.9.19

In a recent sample of 84 used cars sales costs, the sample mean was $6425 with a standard deviation of $3156. Assume the underlying distribution is approximately normal.

  a. Which distribution should you use for this problem? Explain your choice.
  b. Define the Random Variable $\overline{X}$, in words.
  c. Construct a 95% confidence interval for the population average cost of a used car.

     i. State the confidence interval.
     ii. Sketch the graph.
     iii. Calculate the error bound.

  d. Explain what a "95% confidence interval" means for this study.

Exercise 8.9.20

A telephone poll of 1000 adult Americans was reported in an issue of **Time Magazine**. One of the questions asked was "What is the main problem facing the country?" 20% answered "crime". We are interested in the population proportion of adult Americans who feel that crime is the main problem.

  a. Define the Random Variables X and P', in words.
  b. Which distribution should you use for this problem? Explain your choice.
  c. Construct a 95% confidence interval for the population proportion of adult Americans who feel that crime is the main problem.

     i. State the confidence interval.
     ii. Sketch the graph.
     iii. Calculate the error bound.

  d. Suppose we want to lower the sampling error. What is one way to accomplish that?
  e. The sampling error given by Yankelovich Partners, Inc. (which conducted the poll) is $\pm$ 3%. In 1-3 complete sentences, explain what the $\pm$ 3% represents.

Exercise 8.9.21   *(Solution on p. 330.)*
Refer to the above problem. Another question in the poll was "[How much are] you worried
about the quality of education in our schools?" 63% responded "a lot". We are interested in the
population proportion of adult Americans who are worried a lot about the quality of education in
our schools.

1. Define the Random Variables X and P', in words.
2. Which distribution should you use for this problem? Explain your choice.
3. Construct a 95% confidence interval for the population proportion of adult Americans wor-
ried a lot about the quality of education in our schools.

    i. State the confidence interval.
    ii. Sketch the graph.
    iii. Calculate the error bound.

4. The sampling error given by Yankelovich Partners, Inc. (which conducted the poll) is $\pm$ 3%.
In 1-3 complete sentences, explain what the $\pm$ 3% represents.

Exercise 8.9.22
Six different national brands of chocolate chip cookies were randomly selected at the supermarket.
The grams of fat per serving are as follows: 8; 8; 10; 7; 9; 9. Assume the underlying distribution is
approximately normal.

    a. Calculate a 90% confidence interval for the population average grams of fat per serving of
        chocolate chip cookies sold in supermarkets.

        i. State the confidence interval.
        ii. Sketch the graph.
        iii. Calculate the error bound.

    b. If you wanted a smaller error bound while keeping the same level of confidence, what
        should have been changed in the study before it was done?
    c. Go to the store and record the grams of fat per serving of six brands of chocolate chip
        cookies.
    d. Calculate the average.
    e. Is the average within the interval you calculated in part (a)? Did you expect it to be? Why
        or why not?

Exercise 8.9.23
A confidence interval for a proportion is given to be (– 0.22, 0.34). Why doesn't the lower limit of
the confidence interval make practical sense? How should it be changed? Why?

8.9.1 Try these multiple choice questions.

The next three problems refer to the following: According a Field Poll conducted February 8 – 17, 2005,
79% of California adults (actual results are 400 out of 506 surveyed) feel that "education and our schools"
is one of the top issues facing California. We wish to construct a 90% confidence interval for the true
proportion of California adults who feel that education and the schools is one of the top issues facing
California.

Exercise 8.9.24   *(Solution on p. 330.)*
A point estimate for the true population proportion is:

    A. 0.90
    B. 1.27

C. 0.79

D. 400

Exercise 8.9.25   *(Solution on p. 330.)*

A 90% confidence interval for the population proportion is:

A. (0.761, 0.820)

B. (0.125, 0.188)

C. (0.755, 0.826)

D. (0.130, 0.183)

Exercise 8.9.26   *(Solution on p. 330.)*

The error bound is approximately

A. 1.581

B. 0.791

C. 0.059

D. 0.030

The next two problems refer to the following:

A quality control specialist for a restaurant chain takes a random sample of size 12 to check the amount of soda served in the 16 oz. serving size. The sample average is 13.30 with a sample standard deviation is 1.55. Assume the underlying population is normally distributed.

Exercise 8.9.27   *(Solution on p. 330.)*

Find the 95% Confidence Interval for the true population mean for the amount of soda served.

A. (12.42, 14.18)

B. (12.32, 14.29)

C. (12.50, 14.10)

D. Impossible to determine

Exercise 8.9.28   *(Solution on p. 330.)*

What is the error bound?

A. 0.87

B. 1.98

C. 0.99

D. 1.74

Exercise 8.9.29   *(Solution on p. 331.)*

What is meant by the term "90% confident" when constructing a confidence interval for a mean?

A. If we took repeated samples, approximately 90% of the samples would produce the same confidence interval.

B. If we took repeated samples, approximately 90% of the confidence intervals calculated from those samples would contain the sample mean.

C. If we took repeated samples, approximately 90% of the confidence intervals calculated from those samples would contain the true value of the population mean.

D. If we took repeated samples, the sample mean would equal the population mean in approximately 90% of the samples.

The next two problems refer to the following:

Five hundred and eleven (511) homes in a certain southern California community are randomly surveyed to determine if they meet minimal earthquake preparedness recommendations. One hundred seventy-three (173) of the homes surveyed met the minimum recommendations for earthquake preparedness and 338 did not.

Exercise 8.9.30   *(Solution on p. 331.)*

Find the Confidence Interval at the 90% Confidence Level for the true population proportion of southern California community homes meeting at least the minimum recommendations for earthquake preparedness.

   A. (0.2975, 0.3796)
   B. (0.6270, 6959)
   C. (0.3041, 0.3730)
   D. (0.6204, 0.7025)

Exercise 8.9.31   *(Solution on p. 331.)*

The point estimate for the population proportion of homes that do not meet the minimum recommendations for earthquake preparedness is:

   A. 0.6614
   B. 0.3386
   C. 173
   D. 338

8.10 Review[11]

The next three problems refer to the following situation: Suppose that a sample of 15 randomly chosen people were put on a special weight loss diet. The amount of weight lost, in pounds, follows an unknown distribution with mean equal to 12 pounds and standard deviation equal to 3 pounds.

Exercise 8.10.1   *(Solution on p. 331.)*
To find the probability that the average of the 15 people lose no more than 14 pounds, the random variable should be:

   A. The number of people who lost weight on the special weight loss diet
   B. The number of people who were on the diet
   C. The average amount of weight lost by 15 people on the special weight loss diet
   D. The total amount of weight lost by 15 people on the special weight loss diet

Exercise 8.10.2   *(Solution on p. 331.)*
Find the probability asked for in the previous problem.

Exercise 8.10.3   *(Solution on p. 331.)*
Find the 90th percentile for the average amount of weight lost by 15 people.

The next three questions refer to the following situation: The time of occurrence of the first accident during rush-hour traffic at a major intersection is uniformly distributed between the three hour interval 4 p.m. to 7 p.m. Let X = the amount of time (hours) it takes for the first accident to occur.

- So, if an accident occurs at 4 p.m., the amount of time, in hours, it took for the accident to occur is

   _____.

- $\mu = $ _____
- $\sigma^2 = $ _____

Exercise 8.10.4   *(Solution on p. 331.)*
What is the probability that the time of occurrence is within the first half-hour or the last hour of the period from 4 to 7 p.m.?

   A. Cannot be determined from the information given
   B. $\frac{1}{6}$
   C. $\frac{1}{2}$
   D. $\frac{1}{3}$

Exercise 8.10.5   *(Solution on p. 331.)*
The 20th percentile occurs after how many hours?

   A. 0.20
   B. 0.60
   C. 0.50
   D. 1

Exercise 8.10.6   *(Solution on p. 331.)*
Assume Ramon has kept track of the times for the first accidents to occur for 40 different days. Let C = the total cumulative time. Then C follows which distribution?

   A. $U(0,3)$
   B. $Exp\left(\frac{1}{3}\right)$

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[11] This content is available online at <http://cnx.org/content/m16972/1.8/>.

C. $N(60, 30)$
D. $N(1.5, 0.01875)$

Exercise 8.10.7   *(Solution on p. 331.)*
 Using the information in question #6, find the probability that the total time for all first accidents to occur is more than 43 hours.

The next two questions refer to the following situation: The length of time a parent must wait for his children to clean their rooms is uniformly distributed in the time interval from 1 to 15 days.

Exercise 8.10.8   *(Solution on p. 331.)*
How long must a parent expect to wait for his children to clean their rooms?

    A. 8 days
    B. 3 days
    C. 14 days
    D. 6 days

Exercise 8.10.9   *(Solution on p. 331.)*
What is the probability that a parent will wait more than 6 days given that the parent has already waited more than 3 days?

    A. 0.5174
    B. 0.0174
    C. 0.7500
    D. 0.2143

The next five problems refer to the following study: Twenty percent of the students at a local community college live in within five miles of the campus. Thirty percent of the students at the same community college receive some kind of financial aid. Of those who live within five miles of the campus, 75% receive some kind of financial aid.

Exercise 8.10.10   *(Solution on p. 331.)*
Find the probability that a randomly chosen student at the local community college does not live within five miles of the campus.

    A. 80%
    B. 20%
    C. 30%
    D. Cannot be determined

Exercise 8.10.11   *(Solution on p. 331.)*
Find the probability that a randomly chosen student at the local community college lives within five miles of the campus or receives some kind of financial aid.

    A. 50%
    B. 35%
    C. 27.5%
    D. 75%

Exercise 8.10.12   *(Solution on p. 331.)*
Based upon the above information, are living in student housing within five miles of the campus and receiving some kind of financial aid mutually exclusive?

    A. Yes

B. No

C. Cannot be determined

Exercise 8.10.13   *(Solution on p. 331.)*

The interest rate charged on the financial aid is _____ data.

    A. quantitative discrete

    B. quantitative continuous

    C. qualitative discrete

    D. qualitative

Exercise 8.10.14   *(Solution on p. 331.)*

What follows is information about the students who receive financial aid at the local community college.

- 1st quartile = $250
- 2nd quartile = $700
- 3rd quartile = $1200

(These amounts are for the school year.) If a sample of 200 students is taken, how many are expected to receive $250 or more?

    A. 50

    B. 250

    C. 150

    D. Cannot be determined

The next two problems refer to the following information: $P(A) = 0.2$, $P(B) = 0.3$, A and B are independent events.

Exercise 8.10.15   *(Solution on p. 331.)*

$P(A \text{ AND } B) =$

    A. 0.5

    B. 0.6

    C. 0

    D. 0.06

Exercise 8.10.16   *(Solution on p. 331.)*

$P(A \text{ OR } B) =$

    A. 0.56

    B. 0.5

    C. 0.44

    D. 1

Exercise 8.10.17   *(Solution on p. 331.)*

If H and D are mutually exclusive events, $P(H) = 0.25$, $P(D) = 0.15$, then $P(H|D)$

    A. 1

    B. 0

    C. 0.40

    D. 0.0375

8.11 Lab 1: Confidence Interval (Home Costs)[12]

Class Time:

Names:

8.11.1 Student Learning Outcomes:

- The student will calculate the 90% confidence interval for the average cost of a home in the area in which this school is located.
- The student will interpret confidence intervals.
- The student will examine the effects that changing conditions has on the confidence interval.

8.11.2 Collect the Data

Check the Real Estate section in your local newspaper. (Note: many papers only list them one day per week. Also, we will assume that homes come up for sale randomly.) Record the sales prices for 35 randomly selected homes recently listed in the county.

1. Complete the table:

_____	_____	_____	_____	_____
_____	_____	_____	_____	_____
_____	_____	_____	_____	_____
_____	_____	_____	_____	_____
_____	_____	_____	_____	_____
_____	_____	_____	_____	_____
_____	_____	_____	_____	_____

Table 8.2

8.11.3 Describe the Data

1. Compute the following:
    a. $\overline{x} =$
    b. $s_x =$
    c. $n =$
2. Define the Random Variable $\overline{X}$, in words. $\overline{X} =$
3. State the estimated distribution to use. Use both words and symbols.

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[12]This content is available online at <http://cnx.org/content/m16960/1.9/>.

8.11.4 Find the Confidence Interval

1. Calculate the confidence interval and the error bound.

   a. Confidence Interval:
   b. Error Bound:

2. How much area is in both tails (combined)? α =
3. How much area is in each tail? $\frac{\alpha}{2}$ =
4. Fill in the blanks on the graph with the area in each section. Then, fill in the number line with the upper and lower limits of the confidence interval and the sample mean.
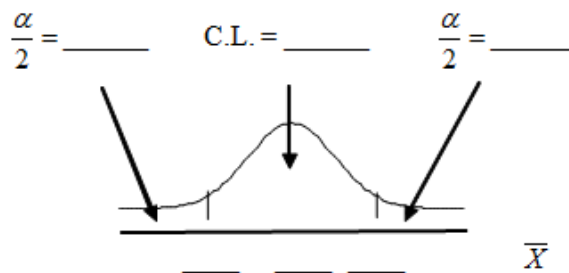
$$\frac{\alpha}{2} = \underline{\qquad} \qquad C.L. = \underline{\qquad} \qquad \frac{\alpha}{2} = \underline{\qquad}$$

$\overline{X}$

Figure 8.5

5. Some students think that a 90% confidence interval contains 90% of the data. Use the list of data on the first page and count how many of the data values lie within the confidence interval. What percent is this? Is this percent close to 90%? Explain why this percent should or should not be close to 90%.

8.11.5 Describe the Confidence Interval

1. In two to three complete sentences, explain what a Confidence Interval means (in general), as if you were talking to someone who has not taken statistics.
2. In one to two complete sentences, explain what this Confidence Interval means for this particular study.

8.11.6 Use the Data to Construct Confidence Intervals

1. Using the above information, construct a confidence interval for each confidence level given.

Confidence level	EBM / Error Bound	Confidence Interval
50%		
80%		
95%		
99%		

Table 8.3

2. What happens to the EBM as the confidence level increases? Does the width of the confidence interval increase or decrease? Explain why this happens.

8.12 Lab 2: Confidence Interval (Place of Birth)[13]

Class Time:

Names:

8.12.1 Student Learning Outcomes:

- The student will calculate the 90% confidence interval for proportion of students in this school that were born in this state.
- The student will interpret confidence intervals.
- The student will examine the effects that changing conditions have on the confidence interval.

8.12.2 Collect the Data

1. Survey the students in your class, asking them if they were born in this state. Let X = the number that were born in this state.

    a. $n =$_____
    b. $x =$_____

2. Define the Random Variable P' in words.
3. State the estimated distribution to use.

8.12.3 Find the Confidence Interval and Error Bound

1. Calculate the confidence interval and the error bound.

    a. Confidence Interval:
    b. Error Bound:

2. How much area is in both tails (combined)? $\alpha=$
3. How much area is in each tail? $\frac{\alpha}{2} =$
4. Fill in the blanks on the graph with the area in each section. Then, fill in the number line with the upper and lower limits of the confidence interval and the sample proportion.
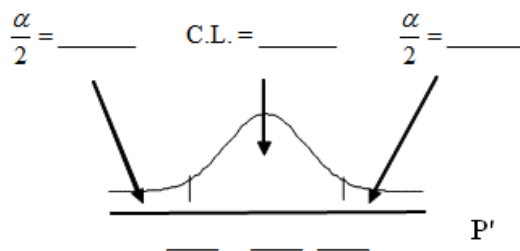

Figure 8.6

8.12.4 Describe the Confidence Interval

1. In two to three complete sentences, explain what a Confidence Interval means (in general), as if you were talking to someone who has not taken statistics.
2. In one to two complete sentences, explain what this Confidence Interval means for this particular study.
3. Using the above information, construct a confidence interval for each given confidence level given.

Confidence level	EBP / Error Bound	Confidence Interval
50%		
80%		
95%		
99%		

Table 8.4

4. What happens to the EBP as the confidence level increases? Does the width of the confidence interval increase or decrease? Explain why this happens.

8.13 Lab 3: Confidence Interval (Womens' Heights)[14]

Class Time:

Names:

8.13.1 Student Learning Outcomes:

- The student will calculate a 90% confidence interval using the given data.
- The student will examine the relationship between the confidence level and the percent of constructed intervals that contain the population average.

8.13.2 Given:

1. **Heights of 100 Women (in Inches)**

59.4	71.6	69.3	65.0	62.9
66.5	61.7	55.2	67.5	67.2
63.8	62.9	63.0	63.9	68.7
65.5	61.9	69.6	58.7	63.4
61.8	60.6	69.8	60.0	64.9
66.1	66.8	60.6	65.6	63.8
61.3	59.2	64.1	59.3	64.9
62.4	63.5	60.9	63.3	66.3
61.5	64.3	62.9	60.6	63.8
58.8	64.9	65.7	62.5	70.9
62.9	63.1	62.2	58.7	64.7
66.0	60.5	64.7	65.4	60.2
65.0	64.1	61.1	65.3	64.6
59.2	61.4	62.0	63.5	61.4
65.5	62.3	65.5	64.7	58.8
66.1	64.9	66.9	57.9	69.8
58.5	63.4	69.2	65.9	62.2
60.0	58.1	62.5	62.4	59.1
66.4	61.2	60.4	58.7	66.7
67.5	63.2	56.6	67.7	62.5

Table 8.5

Listed above are the heights of 100 women. Use a random number generator to randomly select 10 data values.

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[14]This content is available online at <http://cnx.org/content/m16964/1.9/>.

2. Calculate the sample mean and sample standard deviation. Assume that the population standard deviation is known to be 3.3 inches. With these values, construct a 90% confidence interval for your sample of 10 values. Write the confidence interval you obtained in the first space of the table below.
3. Now write your confidence interval on the board. As others in the class write their confidence intervals on the board, copy them into the table below:

90% Confidence Intervals

_____	_____	_____	_____	_____
_____	_____	_____	_____	_____
_____	_____	_____	_____	_____
_____	_____	_____	_____	_____
_____	_____	_____	_____	_____
_____	_____	_____	_____	_____
_____	_____	_____	_____	_____
_____	_____	_____	_____	_____

Table 8.6

8.13.3 Discussion Questions

1. The actual population mean for the 100 heights given above is $\mu = 63.4$. Using the class listing of confidence intervals, count how many of them contain the population mean μ; i.e., for how many intervals does the value of μ lie between the endpoints of the confidence interval?
2. Divide this number by the total number of confidence intervals generated by the class to determine the percent of confidence intervals that contain the mean μ. Write this percent below.
3. Is the percent of confidence intervals that contain the population mean μ close to 90%?
4. Suppose we had generated 100 confidence intervals. What do you think would happen to the percent of confidence intervals that contained the population mean?
5. When we construct a 90% confidence interval, we say that we are **90% confident that the true population mean lies within the confidence interval.** Using complete sentences, explain what we mean by this phrase.
6. Some students think that a 90% confidence interval contains 90% of the data. Use the list of data given (the heights of women) and count how many of the data values lie within the confidence interval that you generated on that page. How many of the 100 data values lie within your confidence interval? What percent is this? Is this percent close to 90%?
7. Explain why it does not make sense to count data values that lie in a confidence interval. Think about the random variable that is being used in the problem.
8. Suppose you obtained the heights of 10 women and calculated a confidence interval from this information. Without knowing the population mean μ, would you have any way of knowing **for certain** if your interval actually contained the value of μ? Explain.

NOTE: *This lab was designed and contributed by Diane Mathios.*

Solutions to Exercises in Chapter 8

Solution to Example 8.1 (p. 289)
$\bar{x} = 7$ and $EBM = 2.5$.
Solution to Example 8.2, Problem 2 (p. 291)

  a. $\bar{x} = 68$
  b. $\sigma = 3$
  c. $n = 36$

Solutions to Practice 1: Confidence Intervals for Averages, Known Population Standard Deviation

Solution to Exercise 8.6.1 (p. 301)
 30.4
Solution to Exercise 8.6.2 (p. 301)
 25
Solution to Exercise 8.6.3 (p. 301)
 σ
Solution to Exercise 8.6.4 (p. 301)
 the age of Winter 2006 Foothill students
Solution to Exercise 8.6.5 (p. 301)
 μ
Solution to Exercise 8.6.6 (p. 301)
 yes
Solution to Exercise 8.6.7 (p. 301)
 Normal
Solution to Exercise 8.6.8 (p. 301)
 0.05
Solution to Exercise 8.6.9 (p. 301)
 0.025
Solution to Exercise 8.6.10 (p. 301)

  a. 24.52
  b. 36.28
  c. 5.88

Solution to Exercise 8.6.11 (p. 302)
$(24.52, 36.28)$

Solutions to Practice 2: Confidence Intervals for Averages, Unknown Population Standard Deviation

Solution to Exercise 8.7.1 (p. 303)

  a. 3.26
  b. 1.02
  c. 39

Solution to Exercise 8.7.2 (p. 303)
 the average number of colors of 39 flags
Solution to Exercise 8.7.3 (p. 303)
 μ

Solution to Exercise 8.7.4 (p. 303)
No
Solution to Exercise 8.7.5 (p. 303)
t_{38}
Solution to Exercise 8.7.6 (p. 304)
0.05
Solution to Exercise 8.7.7 (p. 304)
0.025
Solution to Exercise 8.7.8 (p. 304)

   a. 2.93
   b. 3.59
   c. 0.33

Solution to Exercise 8.7.9 (p. 304)
2.93; 3.59

Solutions to Practice 3: Confidence Intervals for Proportions

Solution to Exercise 8.8.2 (p. 305)
The number of girls, age 8-12, in the beginning ice skating class
Solution to Exercise 8.8.3 (p. 305)

   a. 64
   b. 80
   c. 0.8

Solution to Exercise 8.8.4 (p. 305)
$B(80, 0.80)$
Solution to Exercise 8.8.5 (p. 305)
p'
Solution to Exercise 8.8.6 (p. 305)
The proportion of girls, age 8-12, in the beginning ice skating class.
Solution to Exercise 8.8.8 (p. 305)
0.80
Solution to Exercise 8.8.9 (p. 305)
0.04
Solution to Exercise 8.8.10 (p. 305)

   a. 0.72
   b. 0.88
   c. 0.08

Solution to Exercise 8.8.11 (p. 306)
0.72; 0.88

Solutions to Homework

Solution to Exercise 8.9.1 (p. 307)

   a. i. 71
       ii. 3
       iii. 2.8
       iv. 48

    v. 47

  c. $N\left(71, \frac{3}{\sqrt{48}}\right)$

  d. i. CI: (70.15,71.85)

    iii. EB = 0.85

Solution to Exercise 8.9.3 (p. 307)

  a. i. 8629

    ii. 6944

    iii. 35

    iv. 34

  c. t_{34}

  d. i. CI: (6243, 11,014)

    iii. EB = 2385

  e. It will become smaller

Solution to Exercise 8.9.5 (p. 308)

  a. i. 8

    ii. 4

    iii. 81

    iv. 80

  c. t_{80}

  d. i. CI: (7.12, 8.88)

    iii. EB = 0.88

Solution to Exercise 8.9.7 (p. 309)

  a. i. 2

    ii. 0.1

    iii . 0.12

    iv. 16

    v. 15

  b. the weight of 1 small bag of candies

  c. the average weight of 16 small bags of candies

  d. $N\left(2, \frac{0.1}{\sqrt{16}}\right)$

  e. i. CI: (1.96, 2.04)

    iii. EB = 0.04

  f. i. CI: (1.94, 2.06)

    iii. EB = 0.06

Solution to Exercise 8.9.9 (p. 310)

  a. i. 6

    ii. 3

    iii. 14

    iv. 13

  b. the time for a child to remove his training wheels

  c. the average time for 14 children to remove their training wheels.

  d. t_{13}

  e. i. CI: (3.58, 8.42)

    iii. EB = 2.42

Solution to Exercise 8.9.11 (p. 310)

   a. i. 320
      ii . 400
      iii. 0.80
   c. $N\left(0.80, \sqrt{\frac{(0.80)(0.20)}{400}}\right)$
   d. i. CI: (0.76, 0.84)
      iii. EB = 0.02

Solution to Exercise 8.9.13 (p. 311)

   b. $N\left(0.61, \sqrt{\frac{(0.61)(0.39)}{1200}}\right)$
   c. i. CI: (0.59, 0.63)
      iii. EB = 0.02

Solution to Exercise 8.9.15 (p. 312)

   b. $N\left(0.86, \sqrt{\frac{(0.86)(0.14)}{323}}\right)$
   c. i. CI: (0.8229, 0.8984)
      iii. EB = 0.038

Solution to Exercise 8.9.17 (p. 312)

   a. i. 7.9
      ii. 2.5
      iii. 2.8
      iv. 20
      v. 19
   c. $N\left(7.9, \frac{2.5}{\sqrt{20}}\right)$
   d. i. CI: (6.98, 8.82)
      iii. EB: 0.92

Solution to Exercise 8.9.19 (p. 313)

   a. t_{83}
   b. average cost of 84 used cars
   c. i. CI: (5740.10, 7109.90)
      iii. EB = 684.90

Solution to Exercise 8.9.21 (p. 314)

   b. $N\left(0.63, \sqrt{\frac{(0.63)(0.37)}{1000}}\right)$
   c. i. CI: (0.60, 0.66)
      iii. EB = 0.03

Solution to Exercise 8.9.24 (p. 314)
C
Solution to Exercise 8.9.25 (p. 315)
A
Solution to Exercise 8.9.26 (p. 315)
D
Solution to Exercise 8.9.27 (p. 315)
B

Solution to Exercise 8.9.28 (p. 315)
C
Solution to Exercise 8.9.29 (p. 315)
C
Solution to Exercise 8.9.30 (p. 316)
C
Solution to Exercise 8.9.31 (p. 316)
A

Solutions to Review

Solution to Exercise 8.10.1 (p. 317)
C
Solution to Exercise 8.10.2 (p. 317)
0.9951
Solution to Exercise 8.10.3 (p. 317)
12.99
Solution to Exercise 8.10.4 (p. 317)
C
Solution to Exercise 8.10.5 (p. 317)
B
Solution to Exercise 8.10.6 (p. 317)
C
Solution to Exercise 8.10.7 (p. 318)
0.9990
Solution to Exercise 8.10.8 (p. 318)
A
Solution to Exercise 8.10.9 (p. 318)
C
Solution to Exercise 8.10.10 (p. 318)
A
Solution to Exercise 8.10.11 (p. 318)
B
Solution to Exercise 8.10.12 (p. 318)
B
Solution to Exercise 8.10.13 (p. 319)
B
Solution to Exercise 8.10.14 (p. 319)

   C. 150

Solution to Exercise 8.10.15 (p. 319)
D
Solution to Exercise 8.10.16 (p. 319)
C
Solution to Exercise 8.10.17 (p. 319)
B