2. Basic Structures: Sets, Functions, Sequences, Sums, and Matrices

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ASU
Outline of Chapter 2

1. Sets
2. Set Operations
3. Functions
4. Sequences and Summations
5. Cardinality of Sets
6. Matrices
2.1 Sets

Definition

A **Set** is a well-defined collection of distinct objects, called elements or members of the set. **Well-defined** means that there is a clear rule that enables us to determine whether a given object is an element of the set.

- Notations
  If \( a \) belongs to \( A \), we write \( a \in A \). If not, we write \( a \notin A \).

- Three ways to describe a set
  1. **List(Roaster) method**: all elements are listed between braces. **Ex**: \( A = \{a, b, c\} \)
  2. **Set builder notation**: by stating the property that elements must satisfy. **Ex**: \( \mathbb{R}^+ := \{x \in \mathbb{R} \mid x \geq 0\} \)
  3. **Interval notation**, especially for real numbers. **Ex**: \([a, b]\) or \([a, b)\)
Definition

A = B if two set A and B have the same elements regardless their order.

Example 1

1. \{1, 3, 5\} = \{1, 5, 3\} = \{1, 1, 3, 5, 5\}

Venn Diagram

is convenient to understand relations between sets. The universal set \( U \) is represented by a rectangle. Normally circles are used to represent certain sets and points are used to represent the particular elements of the set.

Example 2

Use to a Venn Diagram to show that if \( A \subset B \) and \( B \subset C \), then \( A \subset C \).

- Notations for the empty (null) set: \( \emptyset \) or \{\}. Don’t be confuse \{\} and \{\emptyset\}. Those are not the same!
**Definition**

The set $A$ is a subset ($A \subseteq B$) if every element of $A$ is also an element of $B$.

- We can use the quantification to define a subset:
  $$\forall x (x \in A \rightarrow x \in B)$$

**Theorem**

For every set $S$, (1) $\emptyset \subseteq S$ (2) $S \subseteq S$.

- When we want to emphasize that $A$ is a subset of $B$ but $A \neq B$, we write $A \subset B$ and say that $A$ is a proper subset of $B$.
  We can also use the quantifications to define a subset:
  $$\forall x (x \in A \rightarrow x \in B) \land \exists x (x \in B \land x \notin A)$$

is true.
Definitions

1. The **cardinality** of a set $S$ means the number of elements in the set $S$. It is denoted by $|S|$ and $|S| = n \geq 0$ for all integers.
2. If $n$ is the finite number, then $S$ is called a finite set. If $S$ is not finite or $(n = \infty)$, then $S$ is called an infinite set.

Example 3

2. $|\emptyset| = 0$
3. $S = \{s \in \mathbb{N} \mid 0 \leq s \leq 5\}$ Then $|S| = 5$
4. $|\mathbb{Z}| = \infty$.

Definitions

1. For a given set $S$, the power set of the set $S$ is the set including all subsets of the set. The power set of $S$ is denoted by $\mathcal{P}(S)$.
2. If $|S| = n$, then $|\mathcal{P}(S)| = 2^n$.

Example 4

1. Let $S = \{0, 1, 2\}$. Then find $\mathcal{P}(S)$ and $|\mathcal{P}(S)|$. 

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Definitions

1. The ordered $n$–tuples $(a_1, a_2, \cdots, a_n)$ is the ordered collection that has $a_1$ as its first element, $a_2$ as its second element, ..., and $a_n$ as its $n$th element. In particular, ordered 2–tuples are called ordered pairs.

2. The **Cartesian product** of $A$ and $B$, denoted by $A \times B$, is the set of all ordered pair $(a, b)$ such that $a \in A$ and $b \in B$. Thus

$$A \times B = \{(a, b) \mid a \in A \land b \in B\}.$$ 

Example 5

1. $A = \{0, 1\}$ and $B = \{x, y, z\}$. Then find $A \times B$.
2. Show that $A \times B \neq B \times A$. 

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